This course will consist of a number of nearly independent chapters devoted to elementary topics from or closely related to partial differential equations. It is intended for students with some background in analysis, including the elements of Lebesgue integration, complex analysis in one variable, linear algebra, and basic functional analysis (classic Banach spaces, uniform boundedness principal).

The aim of the course rather than to follow a shortest straight line path to a certain goal is on the contrary to reveal the many interconnections between subjects. These are often not treated in order to arrive in the fewest steps at some goal. Here my goal is more to reveal interconnectedness and to develop an intuition for basic objects of analysis.

I. How Fourier series should have been discovered I.

Fourier came too early. In particular his discoveries preceded the invention of linear algebra and of complex function theory. In this first topic I will reinvent history showing how one could easily have discovered Fourier Series from its discrete analogue which arises naturally.

In passing we encounter many ideas.

- The Fast Fourier Transform.
- The trapezoid approximation of the integral of a smooth periodic function with mesh size $\delta x = h$ is $O(h^N)$ for all $N$. The proof uses
- The Euler Maclaurin summation formula.
- The Runge phenomenon for polynomial interpolation.
- Fourier interpolation is much better behaved.

The Euler Maclaurin formula and its application to the periodic trapezoid rule is discussed in *Numerical Methods* by B. Dahlquist and A. Bjork and also in *Analyse Numérique et Équations Différentielles* by J.-P. DeMailly. For the Fast Fourier Transform, I recommend the Matlab and recursion based approach of C., Van Loan’s *Introduction to Scientific Computing*.

II. Meet the Laplacian I. Spherical symmetry and the Mean Value
Property.

- Stationarity of Dirichlet integral. Dirichlet integral and surface area.
- Rotational invariance of $\Delta$ and rotation invariant harmonic functions.
- Haar measure on the orthogonal group.
- Symmetry derivation of MVP.
- The laplacian as infinitesimal MVP.
- Orthogonally invariant polynomials. Second proof of MVP.

A brief recap of volume elements and the very easy construction of Haar measure on Lie groups is presented in a handout on the course web page.

III. Meet the Laplacian II. Gravitational field of spherical charge distributions and the MVT.

- Gravitational field of charge distribution. At large distances.
- Gravitational field of a spherical shell, Newton’s Theorem.
- Equal angles of isoceles triangle, symmetry proof.
- Proof of Newton’s Theorem by MVP.

This proof of Newton’s Theorem is given for example in the text *Electricity and Magnetism* by E. Purcell. It is vol II of the Berkeley Physics Series.

IV. Meet the Laplacian III. The five point Laplacian, Discrete heat flow, and Random walk.

- Centered differences and the five point laplacian. Infinitesimal discrete MVP.
- Heat flow on a lattice and the five point laplacian.
- Random walk on the lattice and the semidiscrete heat equation.
- Solution of the discrete Dirichlet Problem by random walk.
- Solution of the Dirichlet Problem by Brownian Motion.

Discrete random walk and the five point laplacian is discussed in §3 of the 1928 paper of Courant Friedrichs Lewy. The paper is posted on the course web page.

V. How Fourier Series should have been discovered II.

Fourier also preceding complex analysis. An independent derivation of Fourier series starting with Laurent expansion in complex analysis and using purely
complex techniques is presented. These tools in hand we derive Shannon’s Sampling Theorem.

VI. Meet the Wave Equation I.
• D’Alembert’s method in 1-d.
• No loss in $L^2$ Sobolev spaces.
• Ill posedness in $L^p$ for $p \neq 2$

VII. Meet the Wave Equation II.
• High frequency asymptotics.
• Rectilinear propagation of light.
• Ill posedness in $L^p$ for $p \neq 2$.

VIII. Stability of difference schemes by Fourier Analysis.

IX. Wave and group velocity for a Klein Gordon example.