MATH 451 HOMOWORK SET 10 (ADDITIONAL)

1. Define

$$f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x+1 & \text{if } x > 0. \end{cases}$$

Determine its inverse function f^{-1} and prove that f^{-1} is continuous at 0.

2. Let $D = [0,1] \cup (2,3]$ and define f by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ x-1 & \text{if } 2 < x \leq 3. \end{cases}$$

Prove that f is continuous on D. Determine f^{-1} and prove that f^{-1} is not continuous on $f(D) := \{f(x) \,|\, x \in D\}$. Does this contradicts Theorem 18.4?

3. Let the function f be a real valued bounded continuous function on $\mathbb{R}$. Prove that there is a solution of the equation

$$f(x) = x, \qquad x \in \mathbb{R}. \tag{0.1}$$

Now choose a number a with $f(a) > a$ and define the sequence (a_n) recursively by defining $a_1 = a$ and $a_{n+1} = f(a_n)$, where $n \in \mathbb{N}$. If f is strictly increasing on $\mathbb{R}$, show that (a_n) converges to a solution of the equation (0.1). This method for approximating the solution is called an *iterative* method.