1. Define 
\[ f(x) = \begin{cases} 
  x & \text{if } x \leq 0 \\
  x+1 & \text{if } x > 0.
\end{cases} \]
Determine its inverse function \( f^{-1} \) and prove that \( f^{-1} \) is continuous at 0.

2. Let \( D = [0,1] \cup (2,3] \) and define \( f \) by 
\[ f(x) = \begin{cases} 
  x & \text{if } 0 \leq x \leq 1 \\
  x-1 & \text{if } 2 < x \leq 3.
\end{cases} \]
Prove that \( f \) is continuous on \( D \). Determine \( f^{-1} \) and prove that \( f^{-1} \) is not continuous on \( f(D) := \{ f(x) \mid x \in D \} \). Does this contradict Theorem 18.4?

3. Let the function \( f \) be a real valued bounded continuous function on \( \mathbb{R} \). Prove that there is a solution of the equation
\[ (0.1) \quad f(x) = x, \quad x \in \mathbb{R}. \]
Now choose a number \( a \) with \( f(a) > a \) and define the sequence \( (a_n) \) recursively by defining \( a_1 = a \) and \( a_{n+1} = f(a_n) \), where \( n \in \mathbb{N} \). If \( f \) is strictly increasing on \( \mathbb{R} \), show that \( (a_n) \) converges to a solution of the equation (0.1). This method for approximating the solution is called an iterative method.