MATH 451 HOMOWORK SET 11 (ADDITIONAL)

1. Assume that f is defined and continuous on $[a, \infty)$; assume further that

$$\lim_{x\to\infty} f(x)$$

exist and is a real number. Show that f is uniformly continuous on $[a, \infty)$.

2. Let $a, b \in \mathbb{R}$. Assume that f, g are uniformly continuous on (a, b), Show that $f + g$, fg are uniformly continuous on (a, b). Can we replace a or b by $-\infty$ or $+\infty$ to get the same conclusions? Prove your assertion or give a counter-example.

3. Let f be a function defined on (a, b). Define

$$\omega_f(\delta) = \sup\{|f(x_1) - f(x_2)| \,\big|\, \forall x_1, x_2 \in (a, b) \text{ satisfying } |x_1 - x_2| < \delta\}.$$

Show that f is uniformly continuous on (a, b) if and only if

$$\lim_{\delta\to 0+} \omega_f(\delta) = 0.$$

ω_f is called the modulus of continuity of the function f.

4. (optional) True or False: a) If a function f is uniformly continuous on intervals $[0, 1]$, and $[1, 2]$. Then f must be uniformly continuous on $[0, 2]$.

b) If a function f is uniformly continuous on intervals $(0, 1)$ and $(1, 2)$. Then f must be uniformly continuous on $(0, 1) \cup (1, 2)$.

5. (optional) Let f be a strictly monotone function on the interval I. Show that f^{-1} is continuous on $f(I)$. (compare with additional problem 1 in homework set 10).