1. Assume that $f$ is defined and continuous on $[a, \infty)$; assume further that 
\[
\lim_{x \to \infty} f(x)
\]
exist and is a real number. Show that $f$ is uniformly continuous on $[a, \infty)$.

2. Let $a, b \in \mathbb{R}$. Assume that $f, g$ are uniformly continuous on $(a, b)$. Show that $f + g, fg$ are uniformly continuous on $(a, b)$. Can we replace $a$ or $b$ by $-\infty$ or $+\infty$ to get the same conclusions? Prove your assertion or give a counter-example.

3. Let $f$ be a function defined on $(a, b)$. Define 
\[
\omega_f(\delta) = \sup \{|f(x_1) - f(x_2)| \mid \forall x_1, x_2 \in (a, b) \text{ satisfying } |x_1 - x_2| < \delta\}.
\]
Show that $f$ is uniformly continuous on $(a, b)$ if and only if 
\[
\lim_{\delta \to 0^+} \omega_f(\delta) = 0.
\]
$\omega_f$ is called the modulus of continuity of the function $f$.

4. (optional) True or False: a) If a function $f$ is uniformly continuous on intervals $[0, 1]$, and $[1, 2]$. Then $f$ must be uniformly continuous on $[0, 2]$.
   b) If a function $f$ is uniformly continuous on intervals $(0, 1)$ and $(1, 2)$. Then $f$ must be uniformly continuous on $(0, 1) \cup (1, 2)$.

5. (optional) Let $f$ be a strictly monotone function on the interval $I$. Show that $f^{-1}$ is continuous on $f(I)$. (compare with additional problem 1 in homework set 10).