

Homework 2

for Wednesday, May 20

- (1) 98 % of all babies survive delivery. However 15 % of all births involve C-sections. When a C-section is performed the baby survives 96% of the time. If a randomly chosen woman does not have a C-section what is the probability that her baby survives?
- (2) Suppose that 5% of men and .25% of women are colorblind. A colorblind person is chosen at random. What is the probability that this person is male? Assume first that there is an equal number of males and females. Then assume that there are twice as many females as males.
- (3) A biased coin has probability p that it lands on heads and probability $q = 1 - p$ that it lands on tails. Suppose it is flipped n times independently.
 - (a) How many elements are in the sample space?
 - (b) What is the probability that at least one head occurs?
 - (c) What is the probability that at least two heads occur?
- (4) Flip a fair coin three times. Let A_{12} be the event that the first and second flip come out the same (i.e. both heads or both tails). Define A_{13} and A_{23} similarly. Are these three events pairwise independent? Are the three events independent?
- (5) An urn contains 5 white and 10 black balls. A fair die is rolled, and that number of balls is randomly chosen from the urn. What is the probability that all the balls selected are white? What is the probability that the die landed on a 3 given that all balls selected are white?
- (6) A parallel system functions whenever at least one of its components works. Consider a parallel system of n components, and suppose that each component independently works with probability $\frac{1}{2}$. Find the probability that the first component works given that the system is functioning.
- (7) There is 50 - 50 chance that the queen carries the gene for hemophilia. If she is a carrier each prince has a 50 - 50 chance of having hemophilia. If the queen has had three princes without the disease, what is the probability that the queen is a carrier? If there is a fourth prince what is the probability that he will have hemophilia?
- (8) For any events E and F show that
$$P(E | E \cup F) \geq P(E | F).$$
- (9) In a simplified model for weather forecasting, suppose that the weather (either wet or dry) tomorrow will be the same as today with probability P . If the weather is dry

on January 1, show that the probability P_n that the weather will be dry n days later satisfies: $P_0 = 1$ and

$$P_n = (2P - 1)P_{n-1} + (1 - P)$$

- (10) The probability of closing the i 'th relay in the circuits shown in the figure below are given by p_i for $i = 1, 2, 3, 4, 5$. If all the relays work independently, what is the probability that current flows between A and B for the circuit?

