(1) Classify all compact connected 1-dimensional manifolds. Recall that our manifolds are Hausdorff by definition.

(2) Let $M$ be an $n$-dimensional manifold. Show that the tangent bundle $TM$ to a manifold $M$ is trivial if and only if there are $n$ vector fields $X_1, \ldots, X_n$ such that for all $p \in M$, the vectors $X_1(p), \ldots, X_n(p)$ are linearly independent.

(3) How many pairs of pants do you have to glue to get a compact surface with $g$ holes for $G \geq 2$?

(4) Show that the alternating $k$-forms on $T_pM$ over a manifold $M$ can be made into a vector bundle over $M$.

(5) (a) Recall that the rank of a linear map is the dimension of the image. Let $M$ and $N$ be two differentiable manifolds, and $f : M \to N$ a differentiable map. Show that the rank of $df_p$ is well defined, i.e. independent of coordinate charts.

(b) Suppose a differentiable map $f : M^n \to N$ has constant rank $k$ on a neighborhood of $f^{-1}(y)$ for some point $y \in N$. Show that $f^{-1}(y)$ is a submanifold of dimension $n - k$ (or is empty).