Homework 3

for Wednesday, May 27

(1) Five Canadians and five Americans are ranked according to their scores on an exam. Assume that all scores are different and that all 10! rankings are equally likely. Let X denote the highest ranking achieved by a Canadian, e.g. if the top scorer is a Canadian, then $X = 1$. Find $P\{X = i\}$ for $i = 1, 2, ..10\}$.

(2) You have \$1,000. Borders stock now sells for \$2 a share. Suppose after one week, the shares will either sell for \$1 or \$4 per share, with equal probability.
   (a) What is your strategy if you want to maximize the amount of money you have at the end of the week (i.e. buy some shares now and sell them after that first week so that you only have money left over).
   (b) What strategy should you follow if you want to maximize the number of shares you own after the first week (so you buy some shares now, and again at the end of the week.

(3) 100 people are having blood tests to see if they have the swine flue. They are tested in groups of 10, their blood gets mixed and tested together. If the test is negative one test suffices for all 10 people. If the pooled test is positive, each individual in that group will be tested individually. Thus for people i such a group, 11 tests will be made total.

Assume that the tests are independent and that the probability that one person has the flue is 10%. Compute the expected number of tests necessary per group.

(4) Suppose the average number of cars abandoned on a highways is 2.2. Approximate the probability that there will be
   (a) No abandoned cars in the next week.
   (b) at least 2 abandoned cars in the next week.

You may assume that this is described by a Poisson distribution.

(5) The probability of being dealt a full house in a hand of poker is approximately .0014. Find an approximation for the probability that in 1000 hands of poker you will be dealt at least two full houses.

(6) Let X be a random variable with probability density function

$$f(x) = a + bx^2, 0 < x < 1$$

and $f(x) = 0$ otherwise.

If $E[X] = \frac{3}{5}$, find a and b. Find the distribution function of X.

(7) Let X be a binomial random variable with parameters n and p. Show that

$$E[\frac{1}{X+1}] = \frac{1-(1-p)^{n+1}}{(n+1)p}$$

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(8) If X has distribution function F what is the distribution function of e^X?

(9) A fair coin in continually flipped until heads appears for the tenth time. Let X denote the number of tails that occur. Compute the probability mass function of X.

(10) A point is chosen at random on a line segment of length L. Interpret this statement and find the probability that the ration of shorter to longer segment is less than $\frac{1}{4}$.