Homework 4

for Friday, June 5

(1) Suppose X is a normal random variable with mean 5. If $P(\{X > 9\}) = .2$, approximate what is $Var(X)$.

(2) In 10,000 independent tosses of a coin, the coin lands heads 5800 times. Is it reasonable to assume that the coin is not fair? Explain.

(3) The number of years that a radio functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Jones buys a used radio what is the probability that it still functions after 8 years?

(4) Each item produced by a company is, independently, of acceptable quality with probability .95. Approximate the probability that at least 10 of the next 150 items produced are unacceptable.

(5) If X is an exponential random variable with parameter λ, and $c > 0$ show that cX is exponential with parameter λ/c.

(6) At a bank, the amount of time a teller spends with a customer is an exponential random variable with mean 5 minutes. If there is a customer in service when you enter the bank, what is the probability that he/she still will be with the teller after an additional 4 minutes?

(7) There are two types of batteries in a bin. When in use, type A (or B) batteries last (in hours) an exponentially distributed time with rate λ (or μ). A battery is randomly chosen from a bin with probability p_A and p_B where $1 = p_A + p_B$. If a randomly chosen battery still operates after t hours of use, what is the probability that it will still be operating after an additional s hours?

(8) Each member of a seven judge panel will independently make a correct decision with probability .7. If the panel's decision is made by majority rule, what is the probability that the panel makes a correct decision?

(9) The joint probability density function of X and Y is given by

$$f(x, y) = c(y^2 - x^2)e^{-y}, -y < x < y, 0 < y < \infty$$

and $f(x, y) = 0$ otherwise.

(a) Find c.

(b) Find the marginal densities of X and Y.

(c) Find $E[X]$.

(10) An ambulance travels back and forth, at a constant speed, along a road of length L. At a certain moment of time, an accident occurs at a point uniformly along the road. Assume also that the ambulance's location at the moment of the accident is uniformly distributed. Assume independence and compute the distribution of its mistaken from the accident.