Homework 5

for Wednesday, June 10

(1) The random variables X and Y have the joint density function
$$f(x,y) = 12x\,y\,(1-x) \text{ for }) < x < 1, 0 < y < 1$$
and equal to 0 otherwise.
(a) Are X and Y independent?
(b) Find $E[X]$.
(c) Find $E[Y]$.
(d) Find $Var(X)$.
(e) Find $Var(X)$.

(2) Three points X_1, X_2, X_3 are selected a t random on a line L of length 1. What is the probability that X_2 lies between X_1 and X_3?

(3) Let X and Y have joint density function
$$f(x,y) = \frac{1}{x^2y^2} \quad x \geq 1, y \geq 1.$$
(a) Compute the joint density function of $U = XY$ and $V = X/Y$.
(b) What are the marginal densities of U and V?

(4) The gross daily sales at Amer's is a normal random variable with mean $\$2,200$ and standard deviation \$230. What is the probability that
(a) that the total gross sales over the next 2 days exceed \$5,000?
(b) That the daily sales exceed \$2,000 in at least 2 of the next 3 days?

(5) Suppose n points $P_1, \ldots, P_n$ are randomly chosen on the perimeter of a circle. What is the probability of the event A that they all lie in a semicircle? Dente by A_i the event that all points lie in the semicircle starting at P_i and going clockwise.
(a) Express A in terms of the A_i.
(b) Are the A_i mutually exclusive?
(c) Find $P(A)$.

(6) The number of people entering a drug store in a given hour is a Poisson random variable with parameter $\lambda = 10$. Compute the conditional probability that at most 3 men entered the drug store in one hour given that at least 10 women entered in that hour. What assumptions have you made?

(7) Our dartboard is a square with sides of length 6. Three circles of radie 1,2, and 3 are all centered at the midpoint of the board. Darts landing in the smallest circle get 30 points, in the annulus adjacent 20 points and the next 10 points. Darts outside that receive 0 points.

Assume that the darts land uniformly distributed in the square. Find the probabilities of the following events:
(a) You score a 20.
(b) You score at least 20.
(c) You score 0.
(d) Find the expected value of your score.

(e) Both your first two throws score at least a 10.
(f) Your total score after two throws is 30.

(8) Two dice are rolled. Let X and Y denote the largest and smallest values obtained. Find the conditional mass function of X given that $Y = i$. Compute it for $i = 1, 2, 3, 4, 5$. Are X and Y independent? Why?

(9) Ben's bowling scores are approximately normally distributed with mean 170 and standard deviation 20, while Tara's are normally distributed with mean 160 and standard deviation 15. If they each bowl one game, assuming their scores are independent random variables, approximate the probability that
(a) Ben's score is higher
(b) their combined score is at least 350.

(10) Let $X_1, \ldots X_2$ be independent exponential random variables having a common parameter λ. Determine the distribution of $\min(X_1, X_2)$. Can you generalize this to n such variables?