(1) Let $M$ be a connected manifold, $p$ and $q$ in $M$. Show that there exists a diffeomorphism $\phi$ such that $\phi(p) = q$. Hint: Might help to use flows.

(2) (a) Let $M$ be a manifold with boundary. Then its boundary $\partial M$ by definition consists of the points which have coordinate charts in to $\bar{H}^n$ and are mapped to the the boundary $\{(x_1, \ldots, x_{n-1}, 0) \}$ of $H^n$. Show that this is well defined and that $\partial M$ is a manifold.

(b) Let $D^2 = \{x \in \mathbb{R}^2 \mid \|x\| \leq 1\}$, called the 2-dimensional disk. Show that $D^2$ is a manifold with boundary.

(3) Let $w \in \mathbb{R}^n$ be a fixed vector. On $\mathbb{R}^n$, define a vector field $X$ by $X(p) = w$. Show that $X$ is still well-defined on $T^n = \mathbb{R}^n / \mathbb{Z}^n$, the $n$-dimensional torus. Find the global flow on the $n$-torus generated by $X$.

(4) Let $A$ be an $n \times n$ matrix. Define a vector field $X(v) := A \cdot v$ on $\mathbb{R}^n$.

(a) Show that the solution curve $c$ for $X$ with initial condition $p$ is given by $c(t) = \exp(tA)p$.

(b) What is the flow generated by $X$?

(c) If $\{v_i\}$ is a basis of eigenvectors of $A$ with (real) eigenvector $\lambda_i$, write the solution curves in terms of the $v_i$.

(5) Let $X$ be a smooth vector field on a compact manifold $M$, an let $\Phi_t : M \rightarrow M$ be its (global) flow. Suppose that $X(p) = 0$ for some point $p \in M$. Show that for all $t$, $\Phi_t(p) = 0$. How about the converse?