The logarithm, and orthogonal matrices

Problem 12. Consider exp as a map from $n \times n$ matrices to $n \times n$ matrices. Show that the derivative of exp at 0 is the $n^2 \times n^2$ identity.

Problem 13. Show that there are open neighborhoods U of 0 and V of Id_n such that exp is a bijection $U \to V$, with smooth inverse.

We define log to be the inverse of exp on some V as above.

Problem 14. Show that $\exp(X^T) = \exp(X)^T$. Show that, if Y is small enough for Y and Y^T to lie in V, then $\log(Y^T) = (\log Y)^T$.

Problem 15. Let $\mathfrak{so}(n)$ be the vector space of skew-symmetric $n \times n$ matrices. Show that $\exp(\mathfrak{so}(n)) \subseteq O(n)$ and there is a neighborhood V of Id_n such that $V \cap O(n) \subseteq \exp(\mathfrak{so}(n))$.