WHAT WE’VE LEARNED THIS YEAR

The exam will concentrate most heavily on Chapters 7 and 8 but will cover the entire term. All of
the following may appear on the exam.

Chapter 1 – Solving linear equations
• Reduced row echelon form
• Using row reduction to solve linear equations

Chapter 2 – Matrices and linear maps
• Basic matrix operations – addition, multiplication, inverse, transpose.
• Injective, surjective, invertible. What these terms mean? How do you test them?
• Using matrices to describe geometry – rotations, reflections, projections, etc.
• Computing the inverse of a matrix by row reduction.

Chapter 3 – Subspaces and dimension
• Definition of a subspace of $\mathbb{R}^n$
• Linearly independent, spanning, basis. What do these words mean? How do you test them?
• Dimension of a subspace.
• Computing a basis for the image or kernel of a linear map.

Chapter 5 – Dot product
• How to compute $\vec{u} \cdot \vec{v}$.
• Geometric interpretation of $\vec{u} \cdot \vec{v}$. Computing lengths of vectors and angles between them.
• What does it mean for a list of vectors to be orthonormal?
• What does it mean for a matrix to be orthogonal?
• What is an orthogonal basis for a subspace? How do you compute it?
• Orthogonal projection – what does it mean and how do you compute it?
• Least squares approximation

Chapter 6 – Determinants
• When is $\det A = 0$, when is $\det A \neq 0$?
• How do operations like adding rows, interchanging rows, rescaling, taking inverse and transpose and multiplying matrices affect determinants?
• Using the properties above to calculate determinants
• Using row reduction to calculate determinants
• Using the sum of patterns formula to calculate determinants
• Geometric meaning of determinant

Chapter 7 – Eigenvalues and Eigenvectors
• What is an eigenvalue? What is an eigenvector? What is the characteristic polynomial?
• How do we compute the characteristic polynomial?
• How do we find eigenvalues?
• How do we find eigenvectors?
• Using eigenvalues to approximate $A^n\vec{v}$ for $n$ large.
• What does it mean to diagonalize a matrix? How do you do it?
• When the characteristic polynomial has complex roots, how do you compute the behavior of $A^n$? What does it look like geometrically?

Chapter 8 – Quadratic forms
• What is the relation between quadratic forms and symmetric matrices?
• What does $x^2 + 2y^2 - 3z^2 = 1$ look like (and other equations like this)?
• What can you say about the eigenvalues of a symmetric matrix? The eigenvectors?
• How can you put a quadratic matrix into diagonal form?
• How do you compute the singular values of a matrix?
• What do singular values say about the geometry of a linear map?