The field axioms

Let $F$ be some set of numbers with operations of addition and multiplication. Then $F$ is called a field if it obeys the following axioms.

**Identity Axioms:** There are elements 0 and 1 of $F$ such that

$$x + 0 = x \quad x \cdot 1 = x$$

for all $x \in F$.

**Inverse Axioms:** For all $x \in F$, there is an element $-x$ such that

$$x + (-x) = 0.$$

If $x$ is a nonzero element of $F$, there is an element $x^{-1}$ such that

$$x \cdot x^{-1} = 1.$$

**Commutativity Axioms:** For all $x$ and $y$ in $F$, we have

$$x + y = y + x \quad x \cdot y = y \cdot x.$$

**Associativity Axioms:** For all $x, y$ and $z$ in $F$, we have

$$x + (y + z) = (x + y) + z \quad x \cdot (y \cdot z) = (x \cdot y) \cdot z.$$

**Distributivity Axiom:** For all $x, y$ and $z$ in $F$, we have

$$x \cdot (y + z) = x \cdot y + x \cdot z.$$

**Nontriviality Axiom:** We have

$$0 \neq 1.$$