Problem 1. Let $G$ be a group in which every element obeys $g^2 = 1$. Show that $G$ is abelian.

Problem 2. Show that an abelian group $A$ of order 100 cannot act faithfully on a set of size 13. By “faithfully”, we mean that the map $A \to S_{13}$ has no kernel.

Problem 3. Let $n$ be a positive integer. Let $G$ be the subgroup of $S_n$ consisting of those maps $\mathbb{Z}/n\mathbb{Z} \to \mathbb{Z}/n\mathbb{Z}$ which are of the form $x \mapsto x + b$ or $x \mapsto -x + b$, for some $b \in \mathbb{Z}/n\mathbb{Z}$. How many conjugacy classes does $G$ have? Your answer should depend on whether $n$ is odd or even.

Problem 4. Let $G$ be a group and let $H$ be a subgroup with $\#(G/H) = n$. Show that there is a normal subgroup $N$ with $N \subseteq H$ with $\#(G/N) \leq n!$.

Problem 5. Does there exist a group $G$ with normal subgroups $N_1$ and $N_2$ such that $N_1 \cong S_5$, $N_2 \cong S_7$, $G/N_1 \cong S_{42}$ and $G/N_2 \cong S_{41}$?

Problem 6. Classify all finite groups $G$ whose only automorphism is the trivial automorphism.