This problem tries to make explicit the computations that are implicit in Hartshorne’s II.6.9. Let $k$ be a field of characteristic not equal to 2 or 3.

Let $X$ be the curve $y^2z = x^3$ in $\mathbb{P}^2_k$ and let $Y$ be the curve $y^2z = x^3 - x^2z$.

(a) Find the smooth points of $X$ and $Y$ respectively. Give an isomorphism between the smooth locus of $X$ and $\mathbb{A}^1 := \text{Spec } k[t]$. Give an isomorphism between the smooth locus of $Y$ and $\mathbb{G}_m := \text{Spec } k[u, u^{-1}]$.

(b) In terms of the parametrizations in (a), give a formula for when three distinct smooth points of $X$ and $Y$ (respectively) are collinear. You should get a nice answer!