First, a warm up problem:

**Problem 8.1.** Suppose that

\[ w = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}. \]

What is \( N_- w \cap wN_+ \)? What open subset of it is \( N_+ B_- \cap (N_- w \cap wN_+) \)?

Recall that we would like to know that \( N_+ \cap B_- wB_- \) is a manifold of dimension \( \ell(w) \).

**Problem 8.2.** Show that

\( N_+ \cap B_- wB_- \cong (N_+ B_- \cap B_- wB_-)/B_- \).

Recall that every element of \( B_- wB_- \) has a unique factorization in the form \( (N_- \cap wN_+ w^{-1})wB_- \) or, equivalently, \( (N_- w \cap wN_+)B_- \).

**Problem 8.3.** Show that

\( (N_+ B_- \cap B_- wB_-)/B_- \cong N_+ B_- \cap (N_- w \cap wN_+) \).

**Problem 8.4.** Show that \( N_+ B_- \cap (N_- w \cap wN_+) \) is an open subset of \( N_- w \cap wN_+ \), and \( N_- w \cap wN_+ \cong \mathbb{R}^{\ell(w)} \).