Proofs Homework Set 7

Math 217 — Winter 2011

Due February 23

Problem 7.1. Let $V$ and $W$ be vector spaces, and suppose that $T : V \to W$ is a one-to-one linear transformation. If there are vectors $v_1, v_2, \ldots, v_k$ in $V$ such that the vectors $T(v_1), T(v_2), \ldots, T(v_k)$ span $W$, prove that the vectors $v_1, v_2, \ldots, v_k$ span $V$.

Problem 7.2. Let $V$ be a vector space. Suppose that $H$ is a nonempty subset of $V$ such that $\text{Span}\{x, y\} \subseteq H$ for all vectors $x, y \in H$. Prove that $H$ is a subspace of $V$.

Problem 7.3. Consider the vector space $C(\mathbb{R})$ of all continuous functions $f : \mathbb{R} \to \mathbb{R}$. Let $Z : C(\mathbb{R}) \to \mathbb{R}$ be defined by $Z(f) = f(0)$.

(a) Prove that $Z$ is a linear transformation.

(b) Prove that $Z$ is onto.

(c) Using part (a), prove that the set $\{f \in C(\mathbb{R}) \mid f(0) = 0\}$ is a subspace of $C(\mathbb{R})$. 