Proofs Homework Set 8

Math 217 — Winter 2011

Due March 9

Problem 8.1. Let $B = \{b_1, b_2, \ldots, b_n\}$ and $C = \{c_1, c_2, \ldots, c_n\}$ be two bases of a vector space $V$. Prove that the coordinate vectors $\{[b_1]_C, [b_2]_C, \ldots, [b_n]_C\}$ form a basis of $\mathbb{R}^n$.

Problem 8.2. Let $U$, $V$, $W$ be three vector spaces and suppose that $T : U \to V$ and $S : V \to W$ are linear isomorphisms (i.e. $T$ and $S$ are one-to-one and onto linear transformations). Prove that their composition $S \circ T$ is also a linear isomorphism. (Recall that the composition $S \circ T$ is the function from $U$ to $W$ defined by $(S \circ T)(x) = S(T(x))$ for all $x \in U$. Don’t forget to show that $S \circ T$ is a linear transformation!)

Problem 8.3. Let $V$ be a subspace of $\mathbb{R}^n$ with dimension $n - 1$ and let $x$ be a vector in $\mathbb{R}^n$ which is not in $V$.

(a) Show that there is a basis $B = \{b_1, b_2, \ldots, b_n\}$ for $\mathbb{R}^n$ such that $\{b_1, \ldots, b_{n-1}\}$ is a basis for $V$ and $b_n = x$.

(b) Use part (a) to show that there is a linear transformation $T : \mathbb{R}^n \to \mathbb{R}$ such that $T(x) = 1$ and the kernel of $T$ is $V$. 