Math 463: Introduction to Mathematical Biology

Computer Lab Assignment 5

PHASE PLANE EXAMPLES

The goal of this lab is for you to practice obtaining numerical solutions to systems of more than two equations. Go to the Handouts section of the course website and download pplane6.m

Alternatively, you can go to the following website and download the version of PPLANE that matches the version of MATLAB that you're using: http://math.rice.edu/˜ *dfield/*.

Note: I have tried pplane7 on my Mac it doesn't seem to work very well, feel free to try it if you wish.

Once you have PPLANE6 on your computer, open MATLAB and type 'pplane6' in the command window. This will open a differential equations editor called PPLANE6 Setup in a new window. The following example problem should already be typed in the fields for x' and y'.

$$\frac{dx}{dt} = 2x - y + 3(x^2 - y^2) + 2xy$$
$$\frac{dy}{dt} = x - 3y - 3(x^2 - y^2) + 3xy$$

- To generate the phase portrait for this system, click on the box that says 'Nullclines' and then hit proceed. This should open a new window showing the phase plane with the nullclines plotted.
- To find steady states, click on 'Solutions' which located on the Menu at the top of this graph. Choose 'Find an equilibrium point' and move the crosshairs that appear to a location of the graph where you think there should be a steady state. PPLANE6 will then plot the steady state nearest to that point with a red circle. Another small window will open and gives you the eigenvalues associated with that steady state.
- To plot the solution trajectories in the phase plane from several different initial values, click on 'Solutions' and choose, 'Plot several solutions'. Move the crosshairs to 7 or 8 different locations in the plane and then hit Return.
- To plot the solution(s) as a function of time, click on 'Graph' and chose the solution you'd like to plot. Cross hairs will appear which you should place on a particular trajectory in the phase plane and hit 'Return'.

1. Play around with this example problem to get comfortable with PPLANE6. Make sure you find all steady states and understand their stability properties. Also be sure you know how to change your window size and can use the 'Graph' command to plot the solution in the x-t, and/or y-t plane You **DO NOT** need to turn this in.

2. Type in the non-dimensional chemostat model and generate the phase portrait for two relevant choices of the parameters. In one case make sure that only one steady state exists and in the other case make sure that both steady states exit. Explain how this phase portrait validates the mathematical results we derived in class.

3. The standard Lotka-Volterra predator-prey model is given below in nondimensional form.

$$\frac{du}{d\tau} = u(1 - v)$$
$$\frac{dv}{d\tau} = \alpha v(u - 1)$$

a) Explain each term in the model equations.
b) Use PPLANE6 to construct a phase portrait for the Lotka-Volterra system by keeping α fixed and using several different initial conditions. What do you notice? Comment on the realism of these results.

c) Now investigate the effect that changing α has on the period of the predator and prey oscillations. To do this graph the predator and prey populations versus time for three very different values of α while keeping the same initial conditions. Estimate the period of the oscillations for each value of α.

4. Now adapt the programs to define and numerically simulate the more realistic predator-prey system below.

$$\frac{du}{d\tau} = u(1 - u) - \frac{auv}{u + d}$$
$$\frac{dv}{d\tau} = bv\left(1 - \frac{v}{u}\right)$$

a) Explain the difference between this model and the previous one. Construct the phase portrait of the system with a = 1, b = 0.05, and d = 0.2 for several different choices of the initial conditions.

b) Plot the predator population vs. time and the prey population vs. time on the same set of axes for one set of initial conditions used in part (a). Explain why these results are more realistic.