1. The interaction between two populations with densities $N_1$ and $N_2$ is modeled by:

$$\frac{dN_1}{dt} = rN_1 \left(1 - \frac{N_1}{K}\right) - aN_1N_2 \left(1 - e^{-bN_1}\right)$$

$$\frac{dN_2}{dt} = -dN_2 + cN_2 \left(1 - e^{-bN_1}\right).$$

where $a, b, c, d, r$ are positive, dimensional parameters.

a) What type of interaction exists between $N_1$ and $N_2$? Describe and interpret each term of the model. Give an explanation in complete sentences.

b) Nondimensionalize the system by writing

$$u = \frac{N_1}{K}, \quad v = \frac{aN_2}{r}, \quad \tau = rt,$$

and defining the new nondimensional parameters

$$\alpha = \frac{c}{r}, \quad \delta = \frac{d}{r}, \quad \beta = bK$$

c) Find the steadys states of the system and describe how their existence and stability change as $\beta$ increases with $0 < \frac{\delta}{\alpha} < 1$.

Hints to Minimize Algebra: When determining the stability of the coexistence steady state, use the criterion that trace of the Jacobian matrix must be negative and the determinant must be positive. Also for the coexistence steady state, DO NOT plug the in the analytical expression for $v$ that you find in part c. Instead plug in $v$ in terms of $u$ and simplify as much as necessary.