Math 463: Intro to Mathematical Biology
Final Project Decision: Due Fridday, October 16th

It's time to start thinking about your final project. The first step is choosing a topic that interests you and a partner to work with. Please turn in a single piece of paper with the subject that you have decided to research and a five references you plan to explore. Also include the name of a classmate who will be working with you. If you'd like some project ideas, please go to the handout section of the Math 463 website.

HOMEWORK # 4: Due Friday, October 16th

1. Write a very short paragraph explaining the assumptions that underlie the logistic grwoth equation (1). You may wish to think about such factors as environmental or individual variability, reproductive ages, and the effect of spatial distributions of the population. Which assumptions are not generally valid?

2. This question also deals with the logistic growth equation:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \quad \text{with} \quad N(0) = N_0 \tag{1}$$

a) Find an analytic expression for the solution of the logistic equation. b) Show that the limit as t approaches infinity of $N(t)$ is equal to K. c) Show that the graph is concave up for $N_0 < N < K/2$ and $N_0 > K$. d) Show that if $K/2 < N < K$, the graph is concave down.

3. Consider the model:

$$\frac{dN}{dt} = f(N) = rKN^2 - rKMN - rN^3 + rMN^2 \quad \text{with} \quad r > 0 \text{ and } 0 < M < K \tag{2}$$

a) Find the per capita growth rate, $g(N)$, and sketch it as a function of N. Write a few sentences explaining how the growth rate of the population changes as the population grows.
b) Find the steady states and determine their stability. What does your analysis suggest about the fate of the population?

4. Models that are commonly used in fisheries are

$$\begin{aligned} \text{Ricker model:} \quad & \frac{dN}{dt} = rNe^{-\beta N} \\ \text{Beverton-Holt model:} \quad & \frac{dN}{dt} = \frac{rN}{\alpha + N} \end{aligned} \tag{3}$$

Analyze the behavior of the solutions to these equations assuming $\alpha, \beta, r > 0$.