

# Math 463: Introduction to Mathematical Biology

## Computer Lab Assignment 4

### The Spruce Budworm Model

1. A model for the spruce budworm population,  $u(t)$ , is given by

$$\frac{du}{dt} = ru \left(1 - \frac{u}{q}\right) - \frac{u^2}{1 + u^2}$$

where  $r$  and  $q$  are positive, dimensionless parameters. The non-zero steady states are thus given by the intersection of the two curves

$$g(u) = r \left(1 - \frac{u}{q}\right) \quad h(u) = \frac{u}{1 + u^2}$$

a) By plotting the functions  $g(u)$  and  $h(u)$  in MATLAB with  $q = 10$ , estimate the values of  $r$  for which the system 'jumps' from 1 to 3 and then back 1 one steady state.

b) You will now use MATLAB to solve the differential equation. First copy the program budworm.m from the course website. Read and understand this program. Using budworm.m and the parameters  $q = 10$  and  $u(0) = u_0 = .1$  estimate the value of  $r$  for which the system moves from the smaller stable steady state to the larger stable steady state.

c) Now change you initial condition to  $u(0) = u_0 = 7.5$  and estimate the value of  $r$  for which the system moves from the larger stable steady state to the smaller stable steady state.

d) Finally, clear all of your previous work and define  $u = 0:.01:9$ . Using  $r = \frac{h(u)}{(1-u/q)}$  with  $q = 10$  plot  $u$  vs.  $r$ . Use this graph and the previous questions to explain what is meant by hysteresis.