The first midterm will cover Chapters 1 and 2, Sections 3.1-3.3 of your textbook (4th edition). It will also cover injectivity and surjectivity of functions. You will not be allowed a calculator, notes, or textbook. Here we briefly review some facts about injective and surjective linear functions.

**Definition.** A linear transformation \( T : \mathbb{R}^m \to \mathbb{R}^n \) is injective if for every \( y \in \mathbb{R}^n \) there is at most one \( x \in \mathbb{R}^m \) such that \( T(x) = y \).

Let \( T : \mathbb{R}^m \to \mathbb{R}^n \) be a linear transformation with associated matrix \( A \). Then \( T \) is injective if and only if \( \text{rref}(A) \) has a pivot in every column if and only if \( A \) has rank \( m \) if and only if the kernel \( \text{ker}(T) \) of \( T \) is 0.

**Definition.** A linear transformation \( T : \mathbb{R}^m \to \mathbb{R}^n \) is surjective if for every \( y \in \mathbb{R}^n \) there is at least one \( x \in \mathbb{R}^m \) such that \( T(x) = y \).

Let \( T : \mathbb{R}^m \to \mathbb{R}^n \) be a linear transformation with associated matrix \( A \). Then \( T \) is surjective if and only if \( \text{rref}(A) \) has a pivot in every row if and only if \( A \) has rank \( n \) if and only if the image \( \text{im}(T) \) is \( \mathbb{R}^n \).

**Definition.** A linear transformation \( T : \mathbb{R}^m \to \mathbb{R}^n \) is invertible if for every \( y \in \mathbb{R}^n \) there is exactly one \( x \in \mathbb{R}^m \) such that \( T(x) = y \).

Let \( T : \mathbb{R}^n \to \mathbb{R}^n \) be a linear transformation with associated matrix \( A \). Then \( T \) is invertible if and only if \( T \) is injective and surjective if and only if the rank of \( A \) is \( n \) if and only if \( \ker(T) = 0 \) if and only if \( \text{im}(T) = \mathbb{R}^n \).

**Exercise:** Determine which of the following matrices determines a linear transformation which is (a) injective, (b) surjective, (c) invertible:

\[
\begin{pmatrix}
1 & -3 & 4 & 7 \\
0 & 1 & 2 & 3 \\
0 & 0 & 0 & 5
\end{pmatrix}
\quad
\begin{pmatrix}
-3 & 5 \\
2 & 2 \\
4 & 0 \\
1 & 0
\end{pmatrix}
\quad
\begin{pmatrix}
1 & -1 & 0 \\
4 & 0 & 2 \\
5 & 5 & 5
\end{pmatrix}
\]