Math 523 Midterm Fall 2001

(1) A company insures a risk given by a Pareto variable with pdf \( f(x) \) given by

\[
f(x) = \frac{1}{2} \left( \frac{10}{x} \right)^6, \quad x \geq 10.
\]

The policy on the risk is stop-loss insurance with deductible \( d \). If \( d = 20 \), and the security loading for the policy is 40%, find the premium the company charges the customer.

(2) An insurance company models annual medical insurance claims from a large corporation by a compound Poisson variable. The mean annual claim frequency is 1200. The size of a claim has mean 250 and standard deviation 1400. The insurance company wants to charge a premium that is large enough so that the probability annual claims exceed the premium is less than 5%. Using the normal approximation, estimate the security loading the insurance company requires on the policy. [Use \( P(Z < 1.645) = 0.95 \) for a standard normal variable \( Z \).]

(3) An aggregate claims variable is compound Poisson. The claim frequency is 4.6 and the individual claims take only the values 1 or 2, with a claim of size 1 having probability 0.4. Find the pure premium on stop-loss insurance with deductible 3.7 for this risk.

(4) A frequency variable \( N \) for an aggregate claims variable is negative binomial with a mean of 4.5 and variance 11.25.
(a) Find the parameters \((r, p)\) for the variable.
(b) Find the probability \( P(N = 1) \).

(5) An insurance company assumes that the aggregate claims variable for a corporate policy is compound Poisson. It has a full credibility standard of 1000 claims for claim frequency. Suppose the severity variable is Gamma with parameters \( r = 1.5, \lambda = 7 \). Assuming the same full credibility standard for premium as for frequency, find the credibility of 1000 claims for premium.

(6) An auto insurance company divides its customers into 2 types, A and B. Type A customers have a probability 1/3 of making a claim in a given year. Type B customers have a probability 1/8 of making a claim in a given year. The probability of a random customer being Type A is 25%. Suppose a customer makes a claim in 2001. Find
(a) The probability the customer will make a claim in 2002,
(b) The probability the customer will make exactly one claim in the 2 year period 2002, 2003.
Assume at most one claim is made in a given calendar year and independence from year to year. Use a Bayesian analysis.