1. Consider the matrix \( A(t) = \begin{pmatrix} t & 0 & t \\ -1 & t^2 & t^3 \\ t & 0 & 1 \end{pmatrix} \).

(1) (5 points) Calculate \( \det(A) \) in terms of \( t \).
(2) (4 points) For what real values of \( t \) is \( A \) invertible?

2. (1) (8 points) Find the general solution of \( y'' + y = \cos(x) \).
(2) (10 points) Find the general solution of \( x' = -x + y, y' = x + y \), you might want to eliminate \( y \) and solve for \( x \), then find \( y \). Here \( y = y(t), x = x(t) \) are differentiable functions of \( t \).

3. Consider the matrix \( A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix} \).

(1)(5 points) Find all eigenvalues, then calculate \( \det(A) \) and \( Tr(A) \).
(2)(8 points) Find a fundamental set of solutions to the system \( X' = AX \).
(3)(8 points) Solve the initial value problem \( X' = AX, X(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \).

4. A forced and damped mass-spring system is governed by the equation \( x'' + 2x' + 26x = 82\cos(4t) \).

(1) (6 points) Find the general solution to the associated homogeneous equation \( x'' + 2x' + 26x = 0 \);
(2) (6 points) Find a particular solution (using method of undetermined coefficients);
(3) (7 points) Write the particular solution in (2) as a steady periodic solution \( x_{sp}(t) = C\cos(wt - a) \), and find this positive constant \( C \) and angle \( a \);
(4) (7 points) If the initial values are given as \( x(0) = 6, x'(0) = 0 \), solve this initial value problem.

5. For a system \( x_1' = x_1 + 5x_3, x_2' = 4x_2 - 3x_3, x_3' = 3x_2 + 4x_3 \) where \( x_1 = x_1(t), x_2 = x_2(t), x_3 = x_3(t) \) are differentiable functions of \( t \).

(1) (5 points) Write this system as a matrix form \( X' = AX \), identify all matrices here;
(2) (6 points) Find all eigenvalues of \( A \).
(3) (8 points) Find a general solution to this system.

6. (7 points) Consider the matrix \( A = \begin{pmatrix} 3 & 1 & 1 \\ -5 & -3 & -1 \\ 5 & 5 & 3 \end{pmatrix} \), to save your time it's known that 2 is an eigenvalue, find all other eigenvalues.