1. (Based on 5.2.54 and 5.2.55) The “Rule of 70” is a method for estimating the doubling times of exponentially growing quantities. If a quantity is growing at a rate of \( p \)% per year, then the “Rule of 70” estimate for the doubling time is \( \frac{70}{p} \) years.

   (a) For parts (i)-(iii) below, answer the question for each of the following following values of \( p \):
   \( p = 0.5, p = 1, p = 5, p = 10, p = 20, p = 50, p = 70, \) and \( p = 100. \)
   (i) Find the “Rule of 70” estimate of the doubling time for a quantity that grows at an annual rate of \( p \)% per year.
   (ii) Find the actual doubling time for a quantity that grows at an annual rate of \( p \)% per year.
   (iii) Suppose $1000 is invested in an account that earns \( p \)% interest per year. Find the actual amount of money in the account at the time given by the “Rule of 70” estimate you found in part (i).

   (b) Based on your observations in part (a), for what kinds of values of \( p \) does it seem that the Rule of 70 gives “good” estimates?

   (c) Suppose a quantity is growing at a continuous rate of \( q \)% per year. As in the initial setup of this problem, assume that the annual growth rate of the quantity is \( p \)%.
   Caution: There is a difference between \( q \) and \( k \) (just as there is a difference between \( p \) and \( r \)).
   (i) Find a formula for \( q \) in terms of \( p \).
   (ii) Find the actual doubling time of the quantity in terms of \( q \).
   (iii) Find the actual doubling time of the quantity in terms of \( p \).

   (d) Use your answer to part (c) to explain why the Rule of 70 works.


3. (Based on 5.R.62) For a little inspiration: http://www.youtube.com/watch?v=fd7D1LWzWmo
   An extremely bouncy rubber ball is dropped onto a parking lot from a helicopter hovering at a height of 80 feet, and, not surprisingly, the ball bounces up and down. Suppose that on each bounce, the ball rises to 85% of the height from which it fell.
   (a) Find the height the ball reaches after it bounces the first time, after the second time, and after the third time.
   (b) Let \( h(n) \) be the height reached by the ball after it bounces off the pavement \( n \) times. Find a formula for \( h(n) \).
   (c) How many bounces will it take before the ball rises to a height of no more than 5 feet? no more than one inch?

Problem 4 is on the next page.
4. \textit{(Based on 5.3.27 – 5.3.30)}

From page 204:

[These problems] use the Richter scale for the strength of an earthquake. The strength, \( W \), of the seismic waves of an earthquake are compared to the strength, \( W_0 \), of the seismic waves of a standard earthquake. The Richter scale rating, \( M \), is

\[
M = \log \left( \frac{W}{W_0} \right).
\]

(a) What is the Richter scale rating of a “standard earthquake”?

(b) What is the Richter scale rating of an earthquake whose seismic waves are twice as strong as “standard seismic waves” (the seismic waves of a “standard earthquake”)?

(c) The 1986 Chernobyl nuclear power plant accident resulted in seismic waves with a Richter scale rating of 3.5. How many times stronger were the Chernobyl seismic waves that “standard seismic waves”?

(d) Suppose two earthquakes of magnitudes \( M_1 \) and \( M_2 \) on the Richter scale have seismic waves of sizes \( W_1 \) and \( W_2 \), respectively. Use properties of logarithms to find a formula for \( M_2 - M_1 \) in terms of \( W_1 \) and \( W_2 \).

(e) Use your formula from part (d) to find the Richter scale rating of an earthquake whose seismic waves are three times as strong as those of the 2008 Sichuan earthquake in China, which had a Richter scale rating of 7.9.

(f) Do Problem 5.3.30 on page 205.