MATH 105 — SECOND UNIFORM EXAM

DEPARTMENT OF MATHEMATICS
University of Michigan

November 15, 2001

NAME: ___________________________ ID NUMBER: ___________________________

SIGNATURE: _______________________

INSTRUCTOR: ______________________ SECTION NO: ___________________________

1. This exam has 9 pages including this cover. There are 10 questions.
2. Use of books, notes, or scratch paper is not allowed. You may certainly use your calculator (but not its manual).
3. Show all of your work! Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. If you are basing your reasoning on a graph, then sketch the graph. Include units in your answers whenever appropriate.
4. One of the skills being tested in this exam is your ability to interpret detailed, precisely worded, directions. Be sure to read the directions carefully and do all that is asked.
5. Stay calm.

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>POINTS</th>
<th>SCORE</th>
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<td>TOTAL</td>
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1. (12 pts.) The function \( f(x) \) is described by the following graph.

\[
\begin{array}{c}
\begin{array}{c}
\text{y} \\
\end{array}

\begin{array}{c}
\begin{array}{c}
3 \\
1 \\
0 \\
\end{array}

\begin{array}{c}
\begin{array}{c}
2\pi \\
\end{array}

\begin{array}{c}
\text{x}
\end{array}
\end{array}
\end{array}
\end{array}
\]

A) \( 2f\left(\frac{x}{2}\right) \)  
B) \( 2f(x) - 2 \)  
C) \( f(2x) \)

D) \( 2f(2x) \)  
E) \( f(-x) \)  
F) \( f(x - \frac{\pi}{2}) \)
2. (6 pts.) The relationship between the intensity of light, \( I \) (in lumens) at a depth of \( x \) feet in Lake Erie is given by

\[
\log\left(\frac{I}{12}\right) = -0.00235x
\]

What is the intensity at a depth of 40 feet? Solve algebraically and note that using your calculator alone will not get you full credit.

3. Consider an increasing exponential function \( f(x) = ab^x \) whose graph has a positive \( y \)-intercept.

(a) (2 pts.) If you multiply \( f(x) \) by the positive stretch factor \( k \), what effect will this have on its \( y \)-intercept? Be specific.

(b) (2 pts.) If you multiply \( f(x) \) by the positive stretch factor \( k \), what effect will this have on its asymptote? Be specific.

(c) (2 pts.) If you multiply \( f(x) \) by the positive stretch factor \( k \), what effect will this have on its average rate of change over any interval? Be specific.
4. Consider the following diagram.

(a) (2 pts.) Which best describes the behavior of \( \sin(\theta) \) as \( P \) moves counterclockwise around the circle from \( A \) to \( B \). (Circle one).

- increasing
- decreasing
- increasing and then decreasing
- decreasing and then increasing
- constant

(b) (2 pts.) Which best describes the behavior of \( \cos(\theta) \) as \( P \) moves counterclockwise around the circle from \( A \) to \( B \). (Circle one).

- increasing
- decreasing
- increasing and then decreasing
- decreasing and then increasing
- constant
5. Consider the functions \( f(x) = x^2 \) and \( g(x) = -x^2 + 10x - 23 \).

(a) (6 pts.) Describe in words the transformations that were used to obtain the graph of \( g(x) \) from that of \( f(x) \).

(b) (4 pts.) Find a formula for \( g(x) \) in terms of \( f(x) \).

(c) (4 pts.) Verify your answer to part (a) by completing the square for \( g(x) \).

(d) (4 pts.) Consider a more general quadratic equation \( j(x) = -(x - h)^2 + k \). What restrictions, if any, on the constants \( h \) and \( k \) will guarantee that \( j(x) \) has exactly two real zeros?
6. A roasted turkey is taken from an oven and placed on a table. Its temperature \( t \) minutes later is given by
\[
H(t) = 75 + 110e^{-0.0128t}
\]
where \( H \) is in \(^{\circ}F\) and \( t \) is in minutes.
(a) (3 pts.) How hot was the turkey when it was removed from the oven?

(b) (4 pts.) How long will it be before the turkey cools to \( 100^{\circ}F \)? (Solve algebraically).

(c) (3 pts.) Which function, \( H(t + 15) \), or \( H(t) + 15 \), approaches the same final temperature as \( H(t) \)?

(d) (3 pts.) What is the temperature of the room in which the table sits?

(e) (2 pts.) Which of the following best describes the graph of \( H(t) \)? (Circle one).

- Increasing and concave up
- Increasing and concave down
- Decreasing and concave up
- Increasing and concave down
7. A small island supports populations of two different species of rabbit, the Fiery Breasted Rabbit (FBR), and the Greater Warbling Rabbit (GWR). The Fiery Breasted Rabbit population presently consists of 104 animals and this population is growing at a continuous rate of 20 percent per year. There are presently three times as many Greater Warbling Rabbits as there are Fiery Breasted Rabbits but the population of Greater Warbling Rabbits is also growing at a continuous rate of 20 percent per year.

(a) (5 pts.) How long will it take for the population of Fiery Breasted Rabbits to double in size?

(b) (3 pts.) By what percentage does the population of Fiery Breasted Rabbits actually increase in a single year?

(c) (5 pts.) Contrast the doubling time of the population of Greater Warbling Rabbits with that of Fiery Breasted Rabbits. What does this suggest about the relationship between the doubling time of a population growing at a continuous growth rate and the initial size of that population?
8. The graph below is a portion of the graph of $\cos(\theta)$.

(a) (4 pts.) What are the coordinates of the point $A$? (Give your answer in terms of $\pi$).

(b) (4 pts.) What are the coordinates of the point $B$? (Give your answer in terms of $\pi$).

9. (12 pts.) For question 9 you do not need to show any work. Circle the appropriate letter.

<table>
<thead>
<tr>
<th>T</th>
<th>F</th>
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<tbody>
<tr>
<td>If $f(x)$ is an even function which is increasing and concave up for $x &gt; 0$, then $f(x)$ is also increasing and concave up for $x &lt; 0$.</td>
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<tr>
<td>The solutions to the equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.</td>
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<tbody>
<tr>
<td>$\sin^{-1} x$ is that number in $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$ whose sine is $x$.</td>
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<tbody>
<tr>
<td>The length of an arc cut off by an angle of $40^\circ$ on a circle of radius 3 inches is 120 inches.</td>
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</table>
10. (6 pts.) The table below contains data which can be modelled by a formula of the form \( f(t) = a \cos(bt) + c \). Find the values for \( a \), \( b \), and \( c \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(t) )</td>
<td>-1</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>-1</td>
<td>2</td>
<td>5</td>
</tr>
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\[ a = \underline{\quad} \]

\[ b = \underline{\quad} \]

\[ c = \underline{\quad} \]