Mathematical Skills and Knowledge for Uniform Exam #1

Recognizing functions from tables and graphs

• If you are given a relationship defined by a table of values, you should be able to determine whether the relationship is a function or not.

• Given a graph, you should be able to apply the vertical line test to determine whether the relationship represented by the graph could be the graph of a function or not.

Understanding statements written in function notation

• Given a statement expressed in function notation, you should be able to identify which symbol(s) represent the value(s) of the input and which symbols represent the value(s) of the output.

• Given a collection of symbols like $f(2)$, you should be able to interpret this as the output value from the function “$f$” corresponding to an input value of “2.”

• If the function gives value of some real-world quantity then you should be able to interpret the meaning of function statements like: $f(2) = 9$ in real-world terms.

• If you are give a function that does not have a formula (e.g. defined by a table or a graph) then you should be able to find the numerical values of statements in function notation and solve equations expressed in function notation. For example, evaluate $f(4)$ or, solve $f(x) = 2$ to find $x$.

• You should be aware of the fact that changes to the input will be represented by changes to the symbols (or the creation of new symbols) that are “inside” the brackets of the function notation.

• You should be aware of the fact that changes to the output will be represented by changes to the symbols (or creation of new symbols) that are “outside” the brackets of the function notation.

• Given a situation that can be described using words or function notation, you should be able to translate written statements about the situation into function notation.
• Given a situation that can be described using words or function notation, you should be able to convert statements expressed in function notation into English sentences that describe the situation.

Using tables and graphs to display information

• You should be thoroughly familiar with drawing graphs and plotting data points by hand. For example:
  
  o Given an ordered pair of numbers, you should know that the first represents an input value that was given to the relationship and that the second represents the output value that the relationship produced.

  o You should know that when you are graphing a relationship between two quantities, the values of the input give the horizontal locations of the points and the values of the output give the vertical locations of the points.

  o You should know that the word "coordinate" or "coordinates" refers to the horizontal and/or vertical location of a point on the graph. For example, the "horizontal coordinate" or "x-coordinate" is the horizontal location of the point; the "vertical coordinate" or "y-coordinate" is the vertical location of the point.

  o If you are given a set of data (e.g. a table of values) and some clues as to which quantity is the input and which is the output, then you should be able to draw a set of axes that will comfortably accommodate all of the data points.

  o If you are given a set of coordinate axes defining a coordinate plane and a point described as an ordered pair or as an entry in a table of data, then you should be able to correctly locate the point on the coordinate plane.

  o You should know that the point where the coordinate axes meet is called the "origin" and has coordinates (0, 0).

  o You should be familiar with the convention that points with negative x-coordinates are plotted to the left of the vertical, and that points with negative y-coordinates are plotted below the horizontal axis.

Linear Functions

• If you are given a table of data, you should be able to determine whether or not the relationship represented by the table is (i) perfectly linear, (ii) able to be approximated by a linear function, or (iii) not well approximated by a perfectly
linear function. For example, you could calculate the slope using several different pairs of points from the table of data and see whether you always get the same number or not for the slope.

- If you are given the equation of a linear function and a specific value of the input, students are able to evaluate the linear function to find the corresponding value of the output.

- If you are given the coordinates two points, you should be able to calculate the slope and the intercept of the linear function whose graph goes through them. If the linear functions represent real-world situations then you should be able to interpret the meaning of the x- and y-coordinates of the intersection point.

- If you are given an accurate picture of the graph of a linear function, you should be able to determine the sign (+ or −) of the slope, m, and intercept, b.

- If you are given the formula for a linear function in a situation in which x and y represent real-world quantities then you should be able to interpret the meaning of the slope, m, and the intercept, b.

- If you are given the equation of a line and the x- and y-coordinates of a point that does not lie on the line, you should be able to calculate the equations of the perpendicular and parallel lines that go through the given point.

**Solving linear equations**

- You should realize that (in addition to being able to plug x-values into equations to get y-values) it is also possible to substitute a known y-value into the equation and calculate the x-value.

- If you have a linear equation and a specific value of y, you should be able to calculate the corresponding value of x. If x and y represent real-world quantities then you should be able to interpret the meaning of the value of x that you calculate.

- If you are given two linear equations make sure that you know how to calculate the x- and y-coordinates of the intersection point. If you know how to do this on a calculator, that’s great, but you also need to know how to find the intersection point by hand.

- If you are given the formula of a linear function you should be able to calculate the x- and y-intercepts of the function. If the quantities x and y represent real-world quantities then you should be able to interpret the meanings of the x- and y-intercepts.
Describing the domain and range of a function

- You should be able to give a definition of the **domain** of a function that is along the following lines: The domain of a function consists of all values that could possibly be plugged into the function.

- You should be able to give a definition of the **range** of a function that is along the following lines: The range of a function consists of all values that could possibly be obtained as outputs from the function.

- Given the graph of a function (possibly including jumps, breaks and "missing" points) you need to be work out the domain of the function and write the domain using inequalities and interval notation.

- Given the graph of a function (possibly including jumps, breaks and "missing" points) you need to be work out the range of the function and write the range using inequalities and interval notation.

- If you are given the formula of a function that includes “fractions” (i.e. one formula over another) square roots (or other powers such as $\frac{1}{2}$, $\frac{1}{4}$, etc.) then you should be able to determine the **domain** of the function using algebra. You should also be able to explain your reasoning process in words.

Functions defined in pieces

- You should be able to recognize situations in which it is not really practical (or easy) to write down a single equation that will represent the whole situation symbolically. (For example, a graph that consists of several disconnected segments.)

- You should be able to handle a situation like this by using several equations – each of which applies over a limited range of $x$-values – to represent the situation symbolically.

- You also have to be careful to remember that when you find a collection of equations to describe a function defined in pieces, each equation that you find only applies over a limited domain.

- If you are given a graph of a function that appears to be made up of basic building blocks such as:
  - Constant functions.
  - Linear functions.
  - Exponential functions.
Quadratic functions.

You need to be able to determine each interval of $x$-values that will require its own individual formula, and then find the formula that applies to each interval.

- Once you have the collection of intervals and formulas, you should be able to write down a formula using the conventional notation for functions defined in pieces. (See page 71 of the textbook for this notation.)

- If you are given a formula for a function defined in pieces, you should be able to draw the graph of the function, including getting the end-points (closed dots or open dots) right.

**Quadratic functions**

- If you see a set of data with a graph that resembles either a "hill" or a "valley," you should know that a quadratic function will do a reasonable job of representing the pattern in the data.

- You should be able to remember that the formula for a quadratic equation in resembles $y = ax^2 + bx + c$, where $a$ is not equal to zero.

- If you are given the equation of a quadratic function, you should be able to predict whether the graph will resemble a "smile" or a "frown," based on the sign (+ or −) of the leading coefficient, $a$.

- If you have a quadratic equation and are given an $x$-value, then you should be able to evaluate the quadratic equation at the $x$-value, respecting the order of operations.

**Solving quadratic equations graphically and algebraically**

- You should know that the idea of "solving" a quadratic equation of the form: $ax^2 + bx + c = 0$ is to find all of the values of $x$ that can be plugged into the left hand side of the equation to produce an output value of zero.

- You should also know that the $x$-values that are "solutions" of the quadratic equation $ax^2 + bx + c = 0$ are also the $x$-values at which the graph of $y = ax^2 + bx + c$ crosses the $x$-axis.

- If you are given a quadratic equation of the form: $ax^2 + bx + c = 0$, you should be able to find all solutions by substituting the values of $a, b$ and $c$ into the quadratic formula:
\[ x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}. \]

**Exponential Functions**

- You need to be able to remember the functional form of the formula for an exponential function:
  \[ f(x) = A \cdot B^x. \]

- If you are given the \(x\)- and \(y\)-coordinates of two points, \((x_1, y_1)\) and \((x_2, y_2)\), you need to be able to calculate the **growth factor**:
  \[ B = \left( \frac{y_2}{y_1} \right)^{\frac{x_1}{x_2-x_1}}. \]

- If you are given the \(x\)- and \(y\)-coordinates of two points you should be able to calculate the equation of the exponential function, \( f(x) = A \cdot B^x \), whose graph passes through the two points.

- If you are given a description of a quantity that contains the following two pieces of information:

  (a) That the quantity always achieves a certain percentage growth, \(r\), during a set period of time, \(T\), and

  (b) The value of the quantity, \(y_0\) at some given time, \(t_0\), you should be able to create an equation for an exponential function to represent this situation by making a table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(t_0)</th>
<th>(t_0 + T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>(y_0)</td>
<td>(y_0 \cdot (1 + r))</td>
</tr>
</tbody>
</table>

and then finding the formula for the exponential function whose graph passes through these two points.

- If you are given a table of data, you should be able to check to see whether or not the data can be represented (at least approximately) using an exponential function by calculating the growth factor using several different pairs of points from the table and checking to see whether or not all of the growth factors obtained are approximately the same or not.

- If you are given an accurate graph of an exponential function you should be able to identify the approximate values of \(A\) (initial value) and \(B\) (growth factor).