Mathematical Skills and Knowledge *Since* Uniform Exam #2

Right Triangle Trigonometry

- If you are given a right-angle triangle and the lengths of two of the sides, you should be able to use the Theorem of Pythagoras to calculate the length of the remaining side.

- If you are given a right-angle triangle and the lengths of two of the sides, you should be able to calculate each of the following quantities (see diagram below):

\[
\sin(\theta), \cos(\theta), \tan(\theta), \sin(\varphi), \cos(\varphi), \tan(\varphi)
\]

\[
\begin{array}{c}
\varphi \\
\theta
\end{array}
\]

- If you are given a right angle triangle together with one of the non-right-angles and the length of one side, you should be able to determine the lengths of the other two sides of the triangle.

- You should be able to solve trigonometric problems that involve some combination of the right-angle trigonometric skills listed above. (Good examples of these kinds of problems are #28 and #29 on page 293.)

Solving Trigonometric Equations

- If you are given a sinusoidal trigonometric function (i.e. one based on either sine or cosine) defined by a formula, you should be able to sketch an accurate graph of the function.

- If you are given a trigonometric equation that follows one of the two patterns shown below (in which \(A\) and \(M\) need not be positive) and an interval of the \(x\)-axis, you should be able to locate all of the approximate solutions of the trigonometric equations.

\[
A \cdot \sin\left(\frac{2\pi}{P} \cdot x\right) + M = k \quad \text{and} \quad A \cdot \cos\left(\frac{2\pi}{P} \cdot x\right) + M = k.
\]
For example, you could draw an accurate sketch of the sinusoidal function over the given interval and a horizontal line at height \( k \). You could then locate the \( x \)-coordinates of any intersection points and give these as the solutions of the trigonometric equation.

- If you are given a pair of sinusoidal trigonometric functions (i.e. based on sine or cosine) that are defined by formulas and an interval, you should be able to find the approximate locations of all points of the interval where the two functions intersect. For example, you could sketch the graphs of both functions over the given interval and then inspect the graph to determine the approximate coordinates of any intersection points.

- If you are given a trigonometric equation that follows one of the two patterns shown below (in which \( A \) and \( M \) need not be positive), you should be able to use algebra and the inverse trigonometric functions on a graphing calculator to find one solution to the trigonometric equation.

\[
A \sin \left( \frac{2\pi}{P} \cdot x \right) + M = k \quad \text{and} \quad A \cos \left( \frac{2\pi}{P} \cdot x \right) + M = k.
\]

For example, you could use algebra to re-arrange the equation to something resembling (I will only give the sine case here – although you should definitely be able to do both the sine and cosine cases): \( x = \frac{P}{2\pi} \cdot \sin^{-1} \left( \frac{k - M}{A} \right) \) and then use the inverse trigonometric functions on your calculator to evaluate this.

- If you are given a trigonometric equation of the form described above (in which \( A \) and \( M \) need not be positive) and an interval of the \( x \)-axis, then you should able to use algebra, inverse trigonometric functions and a well-drawn graph to find all of the solutions of the trigonometric equations that lie in the given interval. For example, you could use the algebraic/calculator process described above to find one solution, and then the symmetries present in the graph to deduce the additional solutions (if any).

- If you are given a trigonometric equation of the form described above (in which \( A \) and \( M \) need not be positive) and asked to find all solutions of the trigonometric equation, then you should able to use algebra, inverse trigonometric functions and a well-drawn graph to find all of the solutions of the trigonometric equation. For example, you could use the algebraic/calculator process described above to find one solution, and then the symmetries present in the graph to deduce the additional solutions (if any), and then add integer multiples of the period, \( P \).
Functions in General

- You should be familiar with all aspects and operations that we have studied in connection with the “function” concept. This includes:
  
  o You should be able to use and modify function notation to describe what is going on in “real-world” situations.
  
  o You should be able to interpret the practical meaning of statements written in function notation (including inverse function notation).
  
  o You should be able to take a relationship (e.g. described by a graph or a table) and check to see whether it is a function or not.
  
  o You should be able to take the formula for a function and sketch an accurate graph of the function.
  
  o You should be able to put together several different functions and several different intervals to create one function that is defined in pieces.
  
  o You should be able to take the formula for a function and determine the domain of the function.
  
  o You should be able to take a function that is defined by a graph and determine the domain and range of the function.

Composition of Functions

- If you are given a pair of functions, say $f$ and $g$, that are defined by tables, and one of the “common” compositions (i.e. $f \circ g$, $g \circ f$, $f \circ f$ or $g \circ g$) then you should be able to write down a table that gives the values of the desired composite (where it is defined).

- If you are given a pair of functions, say $f$ and $g$, that are defined by graphs, and one of the “common” compositions (i.e. $f \circ g$, $g \circ f$, $f \circ f$ or $g \circ g$) then you should be able to draw a graph that shows the graph of the desired composite (where it is defined).

- If you are given a pair of functions, say $f$ and $g$, that are defined by explicit formulas, and one of the “common” compositions (i.e. $f \circ g$, $g \circ f$, $f \circ f$ or $g \circ g$) then you should be able to write down an explicit formula for the desired composite (where it is defined).
• If you are given a composite function defined by an explicit formula, say \( h \), then you should be able to find explicit formulas for two other functions, say \( f \) and \( g \), such that \( h = f \circ g \) and both \( f(x) \neq x \) and \( g(x) \neq x \).

**Combining Functions Using Arithmetic**

• If you are given a pair of functions defined by formulas, you should be able to find formulas for all of the usual arithmetic combinations of the functions (addition, subtraction, multiplication and division).

• If you are given a pair of functions defined by tables, you should be able to construct tables for all of the usual arithmetic combinations of the functions (addition, subtraction, multiplication and division).

• If you are given a pair of functions defined by graphs, you should be able to construct graphs to represent all of the usual arithmetic combinations of the functions (addition, subtraction, multiplication and division).

• If you are given a pair of functions, one defined by a graph and one defined by a formula, you should be able construct graphs to represent all of the usual arithmetic combinations of the functions (addition, subtraction, multiplication and division).

• If you are given a “real world” context that can be described by an arithmetic combination of two functions (e.g. profit = revenue – cost) and a problem described in that context (e.g. what price should a consumer good be sold for in order to maximize profits) you should be able to set up a function that describes the situation, solve an equation or perform some kind of analysis based on the function to solve the problem, and then interpret the meaning of their results in the original “real world” context.

**Inverses**

• You should be able to describe the meaning of the term *invertible function* using something similar to: “A function is an invertible function if the inverse is also a function in its own right.”

• If you are given a function (defined by a table, an explicit formula or a graph) you should be able to use the horizontal line test to determine whether or not the inverse will also be a function in its own right.

• If you are given an invertible function defined by an explicit formula, you should be able to find an explicit formula for the inverse. You should be able to do this
even when the function in a rational function, and when it includes things like square roots, exponentials and logarithms.

- If you are given a function defined by a graph, you should be able to sketch the graph of the inverse.

- If you are given a function defined by a table or a graph, you should be able to find approximate values of the inverse. For example, if given a graph and asked to find the value of \( f^{-1}(10) \) you could find the \( x \)-coordinate of the point that corresponds to a height of 10 on the graph.

- If you are given a “real world” context and a function that relates two quantities in the context, you should be able to give “real world” interpretations of the meaning of statements written in function notation or inverse function notation.

**Power Functions and Power Regression**

- You should able to remember the algebraic form of a power function’s formula and distinguish between the algebraic form of a power function and the algebraic form of an exponential function.

- If you are given a representative portion of the graph of a power function (\( x > 0 \)) you should be able to determine the sign (+ or −) of the constant of proportionality, and determine the approximate sign and magnitude of the power (i.e. negative, between zero and one or greater than one).

- If you are given the coordinates of a pair of points, neither of which includes zero as an \( x \)- or \( y \)-coordinate, you should be able to find the formula of the power function that passes through the two points.

- If you are given a set of data points that can be reasonably well approximated by a power function, you should be able to enter the data points into their calculator and perform a power regression on the data.

- If you are given an equation based on a power function, such as \( k \cdot x^p = c \), then you should be able to use algebra and the laws of exponents to solve for \( x \). In particular, you should not think that using logarithms to solve such a power equation is a good idea.

- If you are given a “real world” situation described by a word problem that can be represented mathematically by a power function, you should be able to use the information contained in the word problem to create a power function (e.g. calculating by hand or using power regression on a calculator), use the power function to solve the problem and then interpret the meaning of the solutions in terms of the original “real world” context.
Polynomial Functions

• You should be able to recognize the algebraic form of a polynomial function.

• If you are given a polynomial function defined by a formula, you should be able to determine the degree of the polynomial, locate the term with the highest power of $x$ and predict the appearance of the graph for large $x$ (positive and negative).

• If you are given a graph, you should be able to locate the points on the graph at which the concavity of the graph changes and refer to these as inflection points.

• If you are given a representative portion of the graph for a non-pathological polynomial function of degree 1, 2, 3 or 4, you should be able to determine the degree of the polynomial function.

• If you are given a set of data points that can be reasonably well approximated by a polynomial function of degree 1, 2, 3 or 4, you should be able to enter the data points into their calculator and perform an appropriate polynomial regression on the data.

• If you are given a polynomial function defined by a formula and the size of an appropriate calculator viewing window, you should be able to use the TRACE (or ZERO or ROOT) feature of a graphing calculator to approximate the roots of a polynomial function.

• If you are given a “real world” situation described by a word problem that can be represented mathematically by a polynomial function, you should be able to use the information contained in the word problem to create a polynomial function (e.g. calculating by hand or using power regression on a calculator), use the polynomial function to solve the problem and then interpret the meaning of the solutions in terms of the original “real world” context.

Short-run Behavior of Polynomial Functions

• If you are given the graph of a polynomial function you should be able to locate the zeros (or roots) of the polynomial.

• If you are given the graph of a polynomial function you should be able to determine the multiplicity (1, 2 or 3) of each root.

• If you are given the graph of a polynomial function that shows all roots of the polynomial and the coordinates of one other point, you should able to find a formula for the polynomial function.
• If you are given a polynomial function defined by a formula and the size of an appropriate calculator viewing window, you should be able to use the TRACE (or MAXIMUM or MINIMUM) feature of a graphing calculator to find the approximate coordinates of the turning points of a polynomial function.

• If you are given a “real world” situation described by a word problem that can be represented mathematically by a polynomial function, you should be able to use the information contained in the word problem to create a polynomial function (e.g. using a diagram and a familiar volume or surface area formula), use the polynomial function to solve the problem and then interpret the meaning of the solutions in terms of the original “real world” context.

Rational Functions and their Graphs

• You should know what a rational function is.

• You should know that the following features of a rational function graph are considered to be “interesting” and know what they look like on a graph:
  o x-intercept(s).
  o y-intercept.
  o Vertical asymptote(s).
  o Horizontal asymptote.
  o Holes in the graph.

• If you are given a rational function defined by a formula (with easy to factor polynomials on top and bottom) you should be able to determine the locations or values of the following features of the rational function’s graph:
  o x-intercept(s).
  o y-intercept.
  o Vertical asymptote(s).
  o Horizontal asymptote.
  o Holes in the graph.

• You should know what a “horizontal asymptote” is and be able to recognize one on a graph.

• If you are given a rational function defined by a formula, you should be able to decide whether or not the rational function’s graph will have a horizontal asymptote.

• If you are given a rational function defined by a formula that does have a horizontal asymptote, you should be able to determine the equation of the horizontal line that the asymptote will approach.
• If you are given a “real world” situation described by a word problem that can be represented mathematically by a rational function, you should be able to use the information contained in the word problem to create a rational function, use the rational function to solve the problem and then interpret the meaning of the solutions in terms of the original “real world” context.

Finding Formulas for Rational Functions

• If you are given the graph of a rational function showing the locations of all “interesting” features (see list below), you should be able to find a formula for the rational function.
  o x-intercept(s).
  o y-intercept.
  o Vertical asymptote(s).
  o Horizontal asymptote.
  o Holes in the graph.

• If you are given the graph of a rational function together with a function $f(x)$ that the graph was obtained from, you should be able to determine the transformations that were applied to $f(x)$ and use these together with the formula for $f(x)$ to create a formula for the rational function.