1. Velocity \( v(t) \) satisfies \( \frac{dv}{dt} = -5, v(0) = \frac{440}{3} \).

\[
v(t) = v(0) + \int_0^t (-5) \, dz = \frac{440}{3} \cdot \left[ 5z \right]_0^t = \frac{440}{3} - 5t
\]

Alternate method: \( v(t) = \int (-5) \, dt = -5t + C \).

Initial velocity \( v(0) = \frac{440}{3} = C \), so \( v(t) = -5t + \frac{440}{3} \).

Height above the lunar surface \( s(t) \) satisfies \( \frac{ds}{dt} = v, s(0) = 0 \).

\[
s(t) = \int_0^t v(z) \, dz = \int_0^t \left( \frac{440}{3} - 5z \right) \, dz = \left[ \frac{440}{3} z - \frac{5}{2} z^2 \right]_0^t = \frac{440}{3} t - \frac{5}{2} t^2
\]

Alternate method: \( s(t) = \int \left( \frac{440}{3} - 5t \right) \, dt = \frac{440}{3} t - \frac{5}{2} t^2 + C \).

Initial height \( s(0) = 0 = C \), so \( s(t) = \frac{440}{3} t - \frac{5}{2} t^2 \).

\( s(t) \) is maximized when \( s'(t) \equiv v(t) = 0 \).

\[
\frac{440}{3} - 5t = 0 \implies t = \frac{440}{15} = \frac{88}{3}. \text{ Then } s \left( \frac{88}{3} \right) = \frac{19360}{9}.
\]

Therefore, the ball reaches maximum height of \( \frac{19360}{9} \) feet after \( \frac{88}{3} \) seconds.

2. (a) \( g(0) = \int_0^0 f(t) \, dt = 0 \), because the upper and lower limits of integration are equal.

\[
g(2) = \int_0^2 f(t) \, dt = \int_0^1 f(t) \, dt + \int_1^2 f(t) \, dt = 0, \text{ because these 2 areas cancel.}
\]

\[
g(4) = g(2) + \int_2^4 f(t) \, dt = \int_2^4 f(t) \, dt = 2, \text{ because this is the area of the triangle}
\]

under the graph of \( f \), so equals \( 0.5 \times \text{base} \times \text{height} = 0.5 \times 2 \times 2 \).

(b) \( g \) is increasing when \( g' > 0 \).

By the Fundamental Theorem of Calculus II, \( g' = f \), so \( g \) is increasing when \( f > 0 \),

which is true when \( x \) is between 1 and 4.

(c) \( g \) is concave down when \( g'' < 0 \), i.e. when \( f' < 0 \), i.e. when \( f \) is decreasing,

which is true when \( x \) is between 0 and 0.5 or between 1.5 and 2.

(d) \( g' = f = 0 \) when \( x = 0, 1, 2 \), so the interior critical points are \( x = 1, 2 \).

Since \( f < 0 \) to the left of 1 and \( f > 0 \) to the right of 1, \( g \) has a local minimum at 1.

Since \( f > 0 \) both to the left and right of 2, \( g \) has neither a local minimum nor a local maximum at 2.

Looking at the endpoints we see that \( g(0) = 0 \) and \( g(4) = 2 \), so the global maximum of \( g \) is 2 and is attained at 4.
3. \( f(x) = e^{\sqrt{x}} = e^{x^{\frac{1}{2}}} \) is concave down between \( x = 0 \) and \( x = 1 \).

Justification: 
\[
 f'(x) = e^{x^{\frac{1}{2}}} \frac{1}{2} x^{-\frac{1}{2}}, \text{ so } f''(x) = e^{x^{\frac{1}{2}}} \frac{1}{4} x^{-1} - e^{x^{\frac{1}{2}}} \frac{1}{4} x^{-\frac{3}{2}} = \frac{1}{4} x^{-1} e^{x^{\frac{1}{2}}} \left[ 1 - x^{-\frac{1}{2}} \right] = \frac{1}{4x} e^{x^{\frac{1}{2}}} \left[ 1 - \frac{1}{\sqrt{x}} \right], \text{ which is negative for } x \text{ between } 0 \text{ and } 1.
\]
Therefore the trapezoidal rule is an underestimate for the integral and the midpoint rule is an overestimate. In other words, \( \text{TRAP}(n) \leq \text{exact integral} \leq \text{MID}(n) \).

Therefore if \( \text{MID}(n) - \text{TRAP}(n) \leq 0.1 \), then the error of using either approximation is not greater than 0.1.

Trying \( n = 2 \) we see that \( \text{MID}(2) = 2.0131 \) and \( \text{TRAP}(2) = 1.9436 \).

\( \text{MID}(2) - \text{TRAP}(2) = 0.0695 < 0.1 \), so for example \( \text{TRAP}(2) = 1.9436 \) is an approximation with error not greater than 0.1.

4. (a) Plot of \( r(t) = 2te^{-\frac{t}{10}} \):

\[
 r(3) = 6e^{-\frac{3}{10}} \approx 4.44490 \text{ ft/yr.}
\]

(b) In the first 3 years the stalk grows 
\[
 \int_{0}^{3} r(t) \, dt = 2 \int_{0}^{3} te^{-\frac{t}{10}} \, dt
\]

In order to find an antiderivative of \( r(t) \) we can use the integration by parts formula 
\[
 \int uv' = uv - \int u'v \text{ with } u = t \text{ and } v' = e^{-\frac{t}{10}}.
\]

In this case \( u' = 1 \) and we may choose \( v = -10e^{-\frac{t}{10}} \), so 
\[
 \int_{0}^{3} r(t) \, dt = 2 \left[ -10te^{-\frac{t}{10}} - \int \left( -10e^{-\frac{t}{10}} \right) \, dt \right]_{0}^{3} = -20 \left[ e^{-\frac{t}{10}} (t + 10) \right]_{0}^{3} = -20 \left( 13e^{-\frac{3}{10}} - 10 \right)
\]
\[
 \approx 7.38726 \text{ ft.}
\]
(c) In the long run, the height will be 
\[ 0.5 + \int_0^\infty r(t) \, dt = 0.5 + \lim_{b \to \infty} \int_0^b r(t) \, dt \]
\[ = 0.5 - 20 \lim_{b \to \infty} \left[ e^{-\frac{t}{10}} (t + 10) \right]_0^b = 0.5 - 20 \lim_{b \to \infty} \left[ e^{-\frac{b}{10}} (b + 10) - 10 \right] \]
\[ = 0.5 - 20(-10) = 200.5 \text{ ft.} \]

5. \[
\int \frac{dx}{x^2 + 2x + 4} = \int \frac{dx}{x^2 + 2x + 1 + 3} = \int \frac{dx}{(x + 1)^2 + (\sqrt{3})^2}
\]
Let \( u = x + 1 \), then \( du = dx \), so \[
\int \frac{dx}{x^2 + 2x + 4} = \int \frac{du}{u^2 + (\sqrt{3})^2}.
\]
Using the given table formula 24 with \( a = \sqrt{3} \) we obtain \[
\int \frac{dx}{x^2 + 2x + 4} = \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{u}{\sqrt{3}} \right) + C = \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x + 1}{\sqrt{3}} \right) + C.
\]

6. If \( x > 1 \), then \( \sqrt{x} > 0 \), so \( x^6 + \sqrt{x} > x^6 > 0 \), so \( 0 < \frac{1}{x^6 + \sqrt{x}} < \frac{1}{x^6} \).

Plots of \( \frac{1}{x^6 + \sqrt{x}} \) and \( \frac{1}{x^6} \):

\[
\begin{array}{cccccccccc}
& 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 & 2.2 & 2.4 & 2.6 & 2.8 & 3 \\
\hline
\frac{1}{x^6 + \sqrt{x}} & 1 & & & & & & & & & & & & & & \\
\frac{1}{x^6} & & & & & & & & & & & & & & & & \\
\end{array}
\]

Therefore, \( 0 \leq \int_1^\infty \frac{dx}{x^6 + \sqrt{x}} \leq \int_1^\infty \frac{dx}{x^6} \).

Since \( \frac{1}{x^6} \) is of the form \( \frac{1}{x^p} \) with \( p = 6 > 1 \), \( \int_1^\infty \frac{dx}{x^6} \) converges.

Therefore, by the comparison test, \( \int_1^\infty \frac{dx}{x^6 + \sqrt{x}} \) converges as well.

In fact, we can be more precise:
\[
\int_1^\infty \frac{dx}{x^6} = \int_1^\infty x^{-6} \, dx = \lim_{b \to \infty} \int_1^b x^{-6} \, dx = \lim_{b \to \infty} \left[ \frac{x^{-5}}{-5} \right]_1^b = \lim_{b \to \infty} \left( -\frac{1}{5b^5} + \frac{1}{5} \right) = \frac{1}{5} = 0.2.
\]

Therefore, \( 0 \leq \int_1^\infty \frac{dx}{x^6 + \sqrt{x}} \leq 0.2 \)