1. (3+2 pts.) The function \( P(t) \) in the graph below represents the total number of pecan pies baked at Abigail’s Bakery up to day \( t \).

![Graph showing the function P(t)]

The rate of production of pies is \( r(t) = dP/dt \).

a. Find the exact value of \( \int_{2}^{3} r(t) \, dt \).

b. What does this value represent in practical terms?

2. (2+2+2 pts.) For each of the antiderivatives below give the name of a technique which can be used to find the antiderivative. If you choose substitution, give a formula for the substitution \( w(x) \) that you would use. If it is integration by parts, give a formula for \( u(x) \) and \( v(x) \) that you would use in the formula \( \int uv' \, dx = uv - \int u'v \, dx \).

1. \( \int \cos^5 x \, \sin x \, dx \)

2. \( \int x \, \cos x \, dx \)

3. \( \int \ln x \, dx \)

3. (3+4 pts.) The Uninspired Investment Fund was worth $120 million at the beginning of 1999. Let \( t \) denote the time in years measured from that time. For one year, the value of the fund measured in millions of dollars increases at a rate

\[
R(t) = 100 e^{-t^3} \text{ million dollars per year.}
\]

a. Using calculus, find an expression for the value \( V(t) \) of the fund at time \( t \) with \( 0 \leq t \leq 1 \).

b. Using appropriate approximate integration methods find upper and lower estimates for the value of the fund at the end of one year so that the exact value is located with an error of no more than 1 million dollars. **Hints.** An upper estimate for a quantity is an approximate value which is guaranteed to be larger than the exact value. A lower estimate is smaller than the exact value. Be sure to specify what methods you are using and why they are upper and lower estimates.
4. (4 pts.) The function \( f(x) \) is given by the graph below.

![Graph of function f(x)](image)

Define a function \( F(x) \) by
\[
F(x) = \int_0^x f(t) \, dt.
\]

Find the exact value of \( F'(1) \). Recall that this notation means the derivative \( F \) when \( x = 1 \). You must give an explanation for your answer in thirty words or less.

5. (5+3 pts.) After a cold front moves in, ice begins forming on the surface of Bluegill Lake. Let \( t \) denote the time measured in hours starting when the ice begins to form. For \( t \geq 0 \) ice forms at a rate given by
\[
\frac{dy}{dt} = k \sqrt{t}.
\]
where \( k \) is a positive constant and the thickness \( y(t) \) is measured in inches.

a. Find an exact formula for the thickness \( y \) as a function of time. The formula should not contain either derivatives or integrals.

b. Safe ice fishing requires at least 10 inches of ice. How long will it take for the lake to be safe for fishing? Your answer is expressed in terms of \( k \).

6. (6+2 pts.) Suppose you are working in a genetics lab that uses radioactive nucleotide tags. There was a spill of this radioactive substance which was absorbed into the floor. The radioactive material decays and radiates into the atmosphere in such a way that the level of radioactivity in the laboratory is
\[
\text{radiation level} = 0.5 e^{-0.015t} \text{ centiGrays per day},
\]
where \( t \) is the number of days since the spill. Recall that the units are defined so that a person in a room with radiation level 1 centiGray per day for a period of \( T \) days accumulates an exposure of \( T \) centiGrays.

a. Find the maximum possible exposure for someone whose lifetime job involves working long hours in the lab. Full credit requires exact rather than approximate evaluations.

b. It is considered safe to accumulate in a lifetime an exposure of as much as 400 centiGrays. Is it safe to work in this lab?

7. (4 pts.) Determine whether the following improper integral converges or diverges. Recall that such an integral is said to converge when it has a finite value, and diverge otherwise.
\[
\int_1^\infty \frac{1}{y + e^y} \, dy
\]