Math 116  Fall 2012

Team Homework 5

Note: To visualize slope fields, you can use the slope field generator at the course website:
http://www.math.lsa.umich.edu/courses/116/Resources/slopefields.html

Problem 1: The logistic differential equation is a common mathematical model of population
growth. In this model, a function \( P(t) \) representing the population size at time \( t \),
satisfies the differential equation
\[
\frac{dP}{dt} = rP \left( 1 - \frac{P}{K} \right),
\]
where \( r \) and \( K \) are constants.

1. Let \( A \) be a constant. Show that the function

\[ P(t) = \frac{K}{1 - Ae^{-rt}} \]

is a solution to the logistic differential equation.

2. The number \( P(t) \) of persons who develop influenza \( t \) days after all members of a
group of 1,000 people have been in contact with a carrier of infection \( (P(0) = 1) \)
satisfies the logistic equation with \( r = 0.39 \) and \( K = 1,000 \).

(a) How many infected individuals are after 2 days?
(b) How many days does it take until 800 are sick?
(c) According to this model, will all 1,000 people eventually get influenza?

Problem 2: Suppose \( B(t) \) is the balance (in dollars) in a bank account earning interest after \( t \)
years the account was open. The initial deposit was 1,000 dlls. If \( B \) satisfies
\[
\frac{dB}{dt} = 0.01B.
\]

1. Use Euler’s method to estimate the balance in the bank account after 1 year using:
   (a) \( \Delta t = 1 \) and 1 step.
   (b) \( \Delta t = 0.5 \) and 2 steps.
   (c) \( \Delta t = 0.25 \) and 4 steps.

2. Use a slope field generator to visualize the slope field of the differential equation
   satisfied by \( B \). Plot your approximations of \( B(t) \) obtained in (1). Are your
   approximations to the value of \( B(1) \) larger or smaller than the actual balance at
   the end of the first year?

Problem 3: Section 11.2: #16.
Problem 4: Let $V(t)$ be the downward velocity (in meters per second) of a falling skydiver $t$ seconds after his parachute opens. The function $V(t)$ satisfies the differential equation

$$\frac{dV}{dt} = g - DV^2 \quad \text{with} \quad V(0) = V_0,$$

where $g = 9.8 \ \text{m/s}^2$ and $D$ is a positive constant.

1. What should be the units of $D$?
2. The slope field of the differential equation satisfied by $V$ is shown below:

Use the slope field to predict the behavior of the downward velocity $V(t)$ of the skydiver for different values of $V_0$. 