Problem 1: A curve is defined by the parametric equations \( x = t^3 - 3t \) and \( y = t^2 \).

i) Find the \( x \) and \( y \) intercepts of the curve.

ii) The curve has two tangents at the point \((0, 3)\). Find the parametric equation of each tangent line.

iii) Find the points on the curve where the tangent is horizontal or vertical.

iv) Find the values of \( t \) where the curve is concave up.

v) Find the length of the loop of the curve.

Problem 2: A projectile is fired from the origin with angle of elevation \( \theta = \frac{\pi}{4} \) radians and initial speed \( v = 10 \, \text{m/s} \). If air resistance is considered, the position of the projectile at time \( t \) in seconds is given by the following parametric equations:

\[
x(t) = \frac{v \cos \theta}{k} (1 - e^{-kt})
\]

\[
y(t) = -\frac{g}{k} t + \left( \frac{v \sin \theta}{k} + \frac{g}{k^2} \right) (1 - e^{-kt}).
\]

where \( g = 9.8 \, \text{m/s}^2 \) and \( k \) is a parameter that accounts for air resistance.

i) At what time will the particle reach its highest point?

ii) What are the coordinates \((x, y)\) of the highest point in the projectile path?

iii) If \( v_x \) is the instantaneous velocity of the projectile in the \( x \)-direction, find \( \lim_{t \to \infty} v_x(t) \).
Problem 3 : Find expressions involving definite integrals that represent the value the following quantities:

i) The equation of the tangent line to the curve \( r = 1 + \cos \theta \) at \( \theta = \frac{\pi}{2} \).

ii) The perimeter of the curve \( r = 1 + \cos \theta \) in the second quadrant.

iii) The area inside the curves \( r = 3 \cos \theta \) and \( r = 1 + \cos \theta \).

Problem 4 : Let \( A \) be the area enclosed between the graph in polar coordinates \( r = 1 + \sin \theta \), the curve in cartesian coordinates \( y = 3x \) and the \( x \)-axis.

i) Find an expression for the value of \( A \) in terms of definite integrals.

ii) Estimate the value of your expression in (i) using \( \text{Left}(4), \text{Right}(4), \text{Mid}(4) \) and \( \text{Trap}(4) \). Write all your sums.

iii) Which of the four approximations in (ii) are guaranteed to be an overestimate for the value of \( A \)? Justify.