Problem 1: Find the exact value of the length of the spiral in polar coordinates $r = e^{-\theta}$ for $\theta \geq 0$.

Problem 2: Let $f(t)$ be a function defined on the interval $[0, \infty)$. The Laplace Transform $L[f]$ of the function $f(t)$ is given by

$$L[f(t)] = \int_0^\infty f(t) e^{-st} \, dt$$

where $s$ is a parameter. Show that

i) $L[t] = \int_0^\infty te^{-st} \, dt = \frac{1}{s^2}$,

ii) $L[e^{3t}] = \int_0^\infty e^{3t} e^{-st} \, dt = \frac{1}{s - 3}$ for $s > 3$.

Problem 3: The average speed of the molecules in an ideal gas is

$$\bar{v} = \frac{4}{\sqrt{\pi}} \left( \frac{M}{2RT} \right)^{\frac{3}{2}} \int_0^\infty v^3 e^{-\frac{Mv^2}{2RT}} \, dv$$

where $M$ is the molecular weight of the gas, $R$ is the gas constant, $T$ is the gas temperature, and $v$ is the molecular speed. Show that

$$\bar{v} = \sqrt{\frac{8RT}{\pi M}}.$$

Problem 4: Project 3 for Chapter 8 (page 461) parts a) through d).