This exam contains 6 problems, worth a total of 100 points. Only work submitted on this booklet will be considered. Please write the answers in the corresponding boxes.

SHOW YOUR WORK.

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<th>Problem</th>
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Problem 1. (3x5= 15 points) This problem consists of three unrelated questions on vector algebra and geometry.

(a) Two vectors \( \vec{v} \) and \( \vec{w} \) are of the form \( \vec{v} = a(-\vec{i} + \vec{j}) \), and \( \vec{w} = b\vec{i} + \vec{j} \), where \( a \) and \( b \) are scalars. If \( \vec{v} + \vec{w} + 10\vec{i} - \vec{j} = \vec{0} \), find the scalars \( a \) and \( b \).

Answer: (a)

(b) A constant force \( \vec{F} = \langle -1, 2, -5 \rangle \) acts on a particle as it moves from \( A(2, 4, 1) \) to \( B(-2, 1, 3) \). Find the work done by the force.

Answer: (b)

(c) Find the area of the parallelogram spanned by the vectors \( \langle 1, 0, -1 \rangle \) and \( \langle -2, 2, 0 \rangle \).

Answer: (c)
Problem 2. (5+5= 10 points) Throughout this problem $\vec{v} = \langle 1, -2, 3 \rangle$ and $P = (2, -1, 4)$.

(a) If $Q = (5, -2, 1)$, find the vector projection of $\vec{PQ}$ onto $\vec{v}$.

(b) Let $\ell$ denote the line through $P$ parallel to $\vec{v}$, and consider the point $Q = (5, -2, 1)$.

Find the point $R$ on $\ell$ such that $\vec{RQ}$ is perpendicular to $\vec{v}$. ($R$ is the orthogonal projection of $Q$ onto $\ell$.)

HINT: Draw a figure; you may want to use part (b).

Answer: (b) $\text{proj}_\vec{v} \vec{PQ} =$

Answer: (c) $R =$
Problem 3.  (7+8+5=20 points) Consider the parametric curve
\[ \vec{r}(t) = \langle t, t^2, t^3 \rangle \]
(a) Find the parametric equation \( \vec{q}(s) \) of the tangent line to the curve at a time \( t = T \).

Answer: (a) \( \vec{q}(s) = \)

(b) If \( T = 2 \), find the point of intersection of the tangent line of part (a) with the \( xy \) plane.

Answer: (b)

(c) Set up an explicit definite integral equal to the length of the arc of the curve from the origin to the point \( (2, 4, 8) \). You do not have to evaluate the integral. Circle your answer.
Problem 4. (7+8 =15 points) Throughout this problem \( f(x, y) = e^{-x}(1 + 4y) \).
(a) Compute the equation of the tangent plane to the graph of the function \( f \) at the point \((0, 2, 9)\).

Answer: (a)

(b) Suppose that the values of \( x \) and \( y \) are known to be within the ranges \( x = 0 \pm 0.2, y = 2 \pm 0.3 \). Using differentials, estimate the corresponding range of possible values of \( z = f(x, y) \).

Answer: (b) \( \leq z \leq \)
Problem 5. (5+5+10=20 points) Throughout this problem
\[ f(x, y, z) = \cos(xy) + z^2 \]

(a) Compute \( \nabla f \) at an arbitrary point \((x, y, z)\).

Answer: (a)

(b) Find the directional derivative of \( f \) at \((1, \pi/2, 1)\) in the direction of the vector \(4\mathbf{i} - 3\mathbf{k}\).

Answer: (b)

(c) Find the equation of the plane tangent to the surface \( \cos(xy) + z^2 = 1 \) at the point \((1, \pi/2, 1)\).

Answer: (c)
Problem 6.  (5x4=20 points)  On the following page you will find five plots.  Assume that the scales of the $x$ and $y$ axes are the same.

You are also given the following statements about plots:

1. This plot represents the graph of the function $f_1(x, y) = \sqrt{x^2 + y^2}$.
2. This is a contour plot of the function $f_1(x, y) = \sqrt{x^2 + y^2}$.
3. These are curves with equations $\sin(x^2 + y^2) = c$, for several evenly-spaced values of the constant $c$.
4. This is a portion of the graph of the function $f_2(x, y) = \sin(x^2 + y^2)$.
5. This is part of the image of the parametric surface $\mathbf{r}(s, t) = (s, t, \sin(s^2 + t^2))$.
6. These are some level curves of the function $f_3(x, y) = x^2 + 2y^2$.
7. This is a portion of the graph of the function $f_4(x, y) = (x^2 - y^2)/(1 + x^2)$.
8. This represents a level surface of the function $g(x, y, z) = z - (x^2 - y^2)/(1 + x^2)$.
9. These are some level curves of the function $f_4(x, y) = (x^2 - y^2)/(1 + x^2)$.
10. This is a portion of the graph of the function $f_5(x, y) = x + y(1 - y)$.
11. These are some level curves of the function $f_5(x, y) = x + y(1 - y)$.
12. This is a portion of the graph of the function $f_6(x, y) = 2 \cos(2x)^2 + y^2$.
13. These are some level curves of the function $f_6(x, y) = 2 \cos(2x)^2 + y^2$.

Fill in the following table with the numbers of the above statements that apply to each plot.

Note: Not all statements apply to some plot, and some plots may have more than one statement that applies to them.

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<th>Plot No.</th>
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