Problem Set 6

Section 15.3, Pages 955–958:

20) Find the first partial derivatives of the function \( f(s, t) = \frac{st^2}{s^2 + t^2} \).

24) Find the first partial derivatives of the function \( f(x, y) = \int_y^x \cos(t^2)dt \).

42) Use implicit differentiation to find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) for \( yz = \ln(x+z) \).

(Visit TEC Visual
http://www.brookscole.com/math_d/templates/student_resources/visuals_modules_5e/visuals/visual_14_3/ to see what this surface looks like.)

44) Same as above but for \( \sin(xyz) = x + 2y + 3z \). (Visit the above URL.)

60) Find partial derivatives \( f_{rss} \) and \( f_{rst} \) for \( f(r, s, t) = r \ln(rs^2t^3) \).

68) Determine whether each of the following functions is a solution of Laplace’s equation \( u_{xx} + u_{yy} = 0 \).

(a) \( u = x^2 + y^2 \),
(b) \( u = x^2 - y^2 \),
(c) \( u = x^3 + 3xy^2 \),
(d) \( u = \ln \sqrt{x^2 + y^2} \),
(e) \( u = \sin x \cosh y + \cos x \sinh y \),
(f) \( u = e^{-x} \cos y - e^{-y} \cos x \).

71) If \( f \) and \( g \) are twice differentiable functions of a single variable, show that the function

\[ u(x, t) = f(x + at) + g(x - at) \]

is a solution of the wave equation \( u_{tt} = a^2 u_{xx} \).
80) The wind-chill index is modeled by the function

\[ W = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16} \]

where \( T \) is the temperature (°C) and \( v \) is the wind speed (km/h). When \( T = -15°C \) and \( v = 30 \text{ km/h} \), by how much would you expect the apparent temperature to drop if the actual temperature decreases by \( 1°C \)? What if the wind speed increases by \( 1 \text{ km/h} \)?

Section 15.4, Pages 966–967:

2) Find an equation of the tangent plane to the surface \( z = 9x^2 + y^2 + 6x - 3y + 5 \) at the point \((1, 2, 18)\).

6) Find an equation of the tangent plane to the surface \( z = e^{x^2-y^2} \) at the point \((1, -1, 1)\).

24) Find the differential of the function \( v = y \cos xy \).

38) Four positive numbers, each less than 50, are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from the rounding.

41) Prove that if \( f \) is a function of two variables that is differentiable at \((a, b)\), then \( f \) is continuous at \((a, b)\).

\text{Hint: Show that}

\[ \lim_{(\Delta x, \Delta y) \to (0,0)} f(a + \Delta x, b + \Delta y) = f(a, b) \]

42) (a) The function

\[ f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases} \]

is graphed in the figure. Using the result of Prob. 41, show that \( f_x(0, 0) \) and \( f_y(0, 0) \) both exist but \( f \) is not differentiable at \((0, 0)\).
(b) Explain why \( f_x \) and \( f_y \) are not continuous at (0, 0).

Section 15.5, Pages 974–976:

6) Use the Chain Rule to find \( dw/dt \) for \( w = xy + yz^2 \), \( x = e^t \), \( y = e^t \sin t \), \( z = e^t \cos t \).

28) Use \( \frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y} = -\frac{F_x}{F_y} \) to find \( dy/dx \) for

\[ y^5 + x^2 y^3 = 1 + ye^{x^2} \]

56) Suppose that the equation \( F(x, y, z) = 0 \) implicitly defines each of the three variables \( x, y, \) and \( z \) as functions of the order two: \( z = f(x, y) \), \( y = g(x, z) \), \( x = h(y, z) \). If \( F \) is differentiable and \( F_x, F_y, \) and \( F_z \) are all nonzero, show that

\[ \frac{\partial z}{\partial x} \frac{\partial x}{\partial y} \frac{\partial y}{\partial z} = -1 \]