Problem Set 3

Section 13.3, Pages 848–850:

10) Find \( a \cdot b \) where \( |a| = 4, |b| = 10 \), the angle between \( a \) and \( b \) is 120\(^\circ\).

41) Show that the vector orth\(_a\)b = b − proj\(_a\)b is orthogonal to \( a \). (It is called an orthogonal projection of \( b \)).

50) If \( r = \langle x, y, z \rangle, a = \langle a_1, a_2, a_3 \rangle \), and \( b = \langle b_1, b_2, b_3 \rangle \), show that the vector equation \( (r - a) \cdot (r - b) = 0 \) represents a sphere, and find its center and radius.

56) Show that all sides of a quadrilateral are equal in length and opposite sides are parallel. Use vector methods to show that the diagonals are perpendicular.

Section 13.4, Pages 856–857:

6) For \( a = i + e^t j + e^{-t}k \) and \( b = 2i + e^t j - e^{-t}k \), find the cross product \( a \times b \) and verify that it is orthogonal to both \( a \) and \( b \).

34) Use the scalar triple product to determine whether the points \( P(1, 0, 1), Q(2, 4, 6), R(3, -1, 2) \) and \( S(6, 2, 8) \) lie in the same plane.

42) Prove part of Theorem 8, that is

\[
 a \times (b \times c) = (a \cdot c)b - (a \cdot b)c. 
\]

Section 13.5, Pages 865–867:

22) Determine whether the lines \( L_1 \) and \( L_2 \) are parallel, skew, or intersecting where \( L_1 \) is defined by \( \frac{x-1}{2} = \)
\[
\frac{y-3}{2} = \frac{z-2}{1} \quad \text{and} \quad L_2 \text{ is defined by } \frac{x-1}{1} = \frac{y-6}{-1} = \frac{z+2}{3}. \quad \text{If they intersect, find the point of intersection.}
\]

30) Find an equation of the plane that contains the line \( x = 3 + 2t, y = t, z = 8 - t \) and is parallel to the plane \( 2x + 4y + 8z = 17 \).

38) Find an equation of the plane that passes through the line of intersection of the planes \( x - z = 1 \) and \( y + 2z = 3 \) and is perpendicular to the plane \( x + y - 2z = 1 \).

68) Find the distance between the parallel planes \( 3x + 6y - 9z = 4 \) and \( x + 2y - 3z = 1 \).

**Extra Problem** Write the equations of rigid body dynamics as a vector equation involving the cross product and as three separate scalar equations. Add in torques in explain how these act on the body.