Mean: \[ E[X] = \frac{1-p}{p} \]

Variance: \[ Var(X) = \frac{1-p}{p^2} \]

Moment GF: \[ M_X(t) = \frac{p}{1-(1-p)e^t}, \quad t \in \mathbb{R}, (1-p)e^t < 1 \]

Properties:
1. If \( X_1 \sim \text{Geometric}[p], \ X_2 \sim \text{Geometric}[p], \) and \( X_1, X_2 \) are independent, then \( X_1 + X_2 \sim \text{NegativeBinomial}[2,p] \)
2. Every geometric distribution has the memoryless property: \( Pr(X > m + n|X > n) = Pr(X > m) \) for all positive integers \( m, n \); moreover, every discrete distribution which satisfies the memoryless property is a geometric distribution for some parameter \( p \)

Truncated Mass Function Bar Charts for Selected Geometric Distributions:

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Remark: Some authors define the geometric distribution to be the number \( Y \) of trials until the first success is obtained, rather than the number of failures. From the definition of \( X \) given above, it is clear that \( Y = X + 1 \); hence any statement about \( X \) can be easily translated into a statement about \( Y \): for example, \( E[Y] = 1/p \). Many discrete random phenomena take values on the non-negative integers, not just the positive integers. Hence, it is preferable to define the special discrete distributions which we intend to use to model such phenomena on the non-negative integers (rather than just the positive integers) wherever possible. It is for this reason that we have defined the geometric distribution to be the number of failures rather than the number of trials.