MATH 513: LINEAR ALGEBRA
ASSIGNMENT 2
HARM DERKSEN

The **Challenging Problems** are due on Friday, September 21 at noon in class. You do **not** have to hand in the routine problems. On a quiz on Monday, September 24, similar problems may appear. It is optional to hand in the **Very Challenging Problems** (but the same deadline applies). These problems will be very hard. You can earn extra credit with the very challenging problems (but they will be graded more strictly).

**Reading**

For Friday, September 14, read Section 5. For Monday, September 17, read Section 6. For Wednesday, September 19, read Section 7.

**Routine Problems**

1. Do Section 3, page 25, Exercise 1(a),(b), and 2(a),(b).
2. Suppose that $F_0$ is a subfield of another field $F$. Show that $F$ has the structure of a vector space over $F_0$. (For example, $\mathbb{C}$ is a vector space over $\mathbb{R}$).
3. Do Section 3, page 25/26, Exercise 7(b), 8(b),(c).
4. Do Section 4, page 32/33, Exercise 1(a),(c),(d),(g) and 3(a),(b),(c),(d),(e),(g),(h).
5. Do Section 4, page 33, Exercise 4(a),(b),(d),(e),(g).

**Challenging Problems**

1. Do Section 4, page 33, exercise 7.
2. Do Section 4, page 33, exercise 8.
3. Show that $P(\mathbb{R})$ the set of polynomial functions on $\mathbb{R}$ is a subspace of $\mathcal{F}(\mathbb{R})$ (the vector space of real-valued functions), then do Exercise 6 of Section 4, page 33.
4. (a) Suppose that \( A, D \) are points on in \( \mathbb{R}^2 \). Let \( Q \) be the point on the line segment \( AD \) such that \( \overrightarrow{AQ} = 2\overrightarrow{QD} \). Prove that \( Q = \frac{1}{3}A + \frac{2}{3}D \).

(b) Suppose that \( A, B, C \) form the vertices of a triangle. Let \( D \) be the midpoint of \( BC \), let \( E \) be the midpoint of \( AC \) and let \( F \) be the midpoint of \( AB \). Prove that the line segments \( AD, BE \) and \( CF \) meet in one point.

\( \text{(Hint: Take } Q \text{ on } AD \text{ such that } \overrightarrow{AQ} = 2\overrightarrow{QD}, R \text{ on } BE \text{ such that } \overrightarrow{BR} = 2\overrightarrow{RE} \text{ and } S \text{ on } CF \text{ such that } \overrightarrow{CS} = 2\overrightarrow{SF} \text{ and prove that } Q = R = S. \)"

**Very Challenging Problems**

1. In a vector space, an "infinite" set of vectors

\[
\{a_1, a_2, a_3, a_4, \ldots \}
\]

is said to be linearly independent if any finite subset of vectors is linearly independent. Show that the set

\[
\{\cos(x), \sin(x), \cos(2x), \sin(2x), \cos(3x), \sin(3x), \ldots \}
\]

is a linearly independent set of "vectors" in the \((\mathbb{R})\)-vector space \( \mathcal{F}(\mathbb{R}) \) of real valued functions on \( \mathbb{R} \).