PROBLEM SET 4: BOUND

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1. (a) Show that $A_2(4, 3) \leq 2$ (by inspection, for example).
   (b) Show that $A_2(n, 3) \leq 2^{n-3}$ for all $n$. This improves the Singleton Bound by a factor 2.
   (c) Determine $A_2(4, 3), A_2(5, 3), A_2(6, 3), A_2(7, 3)$.
   (d) As described in Van Lint at the end of §4.4, there exists a $(10, 40, 4)$-code. Show that $A_2(8,3) \geq 10$ and $A_2(9,3) \geq 20$.

2. Show that a Hadamard code $(n, 2n, \frac{1}{2}n)$ is optimal. (Show first that after shortening it is an optimal code).

3. Write down the linear programming problem one has to solve in order to obtain the linear programming upper bound for $A_2(10, 4)$. (For extra credit: solve it! You probably need a computer to maximize. For people using maple, there is a package called ‘simplex’ and a command called ‘maximize’ which can be used to solve this type of linear programming problems.)

4. Compare all asymptotic lower and upper bounds to estimate $\alpha(\frac{1}{3})$ (for binary codes). What are the sharpest lower and upper bounds you can find?

5. Give an upper bound for $A_2(20, 5)$. The student with the sharpest correct upper bound wins a chocolat bar. (There exists a $(20, 2560, 5)$-code, so this gives a lower bound for $A_2(20, 5)$).