Problem 1: Consider the following

\[ \frac{d^2x}{dt^2} + \alpha x = \Gamma \cos \omega t \]

The solution to this equation was provided in class. Run simulations for this equation by varying \( \omega \) for some fixed \( \alpha \) and consider conditions when \( \omega^2 \to \alpha \). What do you noticing happening to the solution as you get closer to \( \alpha \)?

Problem 2: Now introduce damping, using a constant damping coefficient, \( k \). What does the equation become? Vary both \( k \) and \( \omega \) and explain the differences between the solutions in 1 and the solutions you are now seeing. What is different about the solution? What happens as \( \omega^2 \to \alpha \)?

Problem 3: To introduce the effects of nonlinearity on resonance consider an example of the Duffing equation with no damping and a weak non-linearity.

\[ \frac{d^2x}{dt^2} + \Omega^2 x - \epsilon x^3 = \Gamma \cos t \]

In this case we simply let \( \omega = 1 \). Find the order one solution for the equation and the order \( \epsilon \) solution. Consider the periodicity condition \( x(\epsilon, t + 2\pi) = x(\epsilon, t) \) What does the solution tell you about resonance? Run some simulations for \( \Omega \) near resonance.

Problem 4: Now consider the full Duffing equation,

\[ \frac{d^2x}{dt^2} + k \frac{dx}{dt} + \alpha x - \delta x^3 = \Gamma \cos \omega t \]

Run simulations for varying parameters and explain what you are seeing? Try at least three different cases.