Assignment #1. Due 14 January.

1. Let $A$ denote an $N \times N$ matrix and $e := (1, 0, \ldots, 0)$ the first elementary basis vector of $\mathbb{R}^N$. Find the relation between the vectors $Ae$, $A^T e$, and the matrix $A$.

2. a.) Show that a permutation matrix is orthogonal. b.) Give an example of a $4 \times 4$ permutation matrix whose eigenvalues are the four complex numbers $\pm 1, \pm i$. Discussion. In particular this shows that the eigenvalues of an orthogonal matrix are not necessarily real.

3. Ciarlet problem 1.1-5 on page 8. In future we will use the shorthand $8/1.1-5$.

4. Consider the 2-point boundary value problem

$$-rac{d^2 \phi(x)}{dx^2} + x \phi(x) = (1 + 2x - x^2) e^x, \quad \text{for} \quad 0 \leq x \leq 1,$$

$$\phi(0) = 1, \quad \phi(1) = 0.$$  

Verify that $\phi(x) = (1 - x)e^x$ is a solution. We will later show that this is the only solution. This is a consequence of the fact that the coefficient of $\phi$ which Ciarlet calls $c(x)$ is in this case equal to $x$ which is nonnegative on the interval $[0, 1]$.

Write a computer program to solve the problem using the second order finite difference scheme discussed in class and in §3.1 of Ciarlet. Solve the resulting tridiagonal system using the $LU$ factorization. Run the code for $h = 2^{-p}$, $p = 1, 2, \ldots, 12$. Display the results in the same format as Table 3.1-1 on page 74 of Ciarlet’s book. Discuss the behavior of the error as $h$ decreases. Discussion. You should discern the order of accuracy of the method and something else too.