Midterm 2

due by November 22, 2000

Name:

Explain your arguments as carefully and clearly as possible.

1. Let $X, Y$ and $Z$ be intervals. Suppose $f : X \mapsto Y$ and $g : Y \mapsto Z$ are uniformly continuous. Show that $g \circ f$ is uniformly continuous.

2. Let $f(x) = \sum_{k=0}^{2n} a_k x^k$ be a polynomial of even degree $> 0$. Suppose $a_{2n} > 0$, and that $\min \{ f(x) \mid x \in \mathbb{R} \} < 0$. Show that $f$ has at least two zeroes, i.e. there are at least two distinct $x$ with $f(x) = 0$.

3. Let $I_n \neq \emptyset$ be a sequence of closed intervals such that $I_{n+1} \subset I_n$. Show that $\cap_{n=1}^{\infty} I_n \neq \emptyset$.

4. Let $f_n$ be a sequence of continuous functions on a closed interval $I$ converging uniformly to $f$. Is it true that the $\max \{ f_n(x) \mid x \in I \}$ converges to $\max \{ f(x) \mid x \in I \}$? Prove your claim.

5. Show that if $\sum_{k=0}^{\infty} | a_k | < \infty$. Show that $\sum_{k=0}^{N} a_k x^k$ converges uniformly to a continuous function on $[-1, 1]$.

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